A CANADIAN HIGH SCHOOL ARITHMETIC

STANLEY AND SMITH

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AUTHORIZED BY THE MINISTER OF EDUCATION
EDMONTON, ALBERTA

TORONTO
OXFORD UNIVERSITY PRESS
1925

PRINTED IN CANADA
T. H. BEST PRINTING CO., LIMITED
TORONTO, ONT.

PREFACE

It is assumed in this treatment of the subject that Arithmetic is primarily a matter of accurate calculation. No attempt is made to go into legal or industrial technicalities. Of necessity however, much useful knowledge of certain things, processes and transactions must be obtained before the calculations to be made can be intelligently interpreted. In this knowledge, and in the ability to make accurately the calculations involved in the problems of every day life lies, very largely, the practical value of Arithmetic.

On the other hand, this book is intended to give high school students in their last course in Arithmetic such an acquaintance with its matter and processes, as will place it on a strictly scientific basis and give it its proper place among the Mathematical Sciences. This should be of special value to students intending to enter the Normal School or to those preparing for the University and work in Higher Mathematics. With this in mind, a considerable amount of matter that has not generally been found in text books in Arithmetic is included in this text.

The habit of constant checking in mathematical calculations is encouraged throughout the first chapter and includes explanations of tests for accuracy for the simple rules, and of the method of checking by approximations. Algebraic solutions are advised in certain types of problems, inasmuch as the purpose of Algebra is thus in a measure fulfilled and the recognition of the practical uses of the equation adds to the student's mathematical power.

The inclusion of enough simple interest tables to provide for considerable practice in their use is almost a necessity in a modern High School Arithmetic. "Samples" either of these or of logarithmic tables are quite inadequate.

Graphs and elementary logarithmic calculations are included as properly belonging to the subject of Arithmetic. For the majority of students it is the graph of Arithmetic rather than that of Algebra that has, or will have, a practical meaning. In logarithms, Arithmetic draws on Algebra for its foundations, but logarithms belong to Arithmetic rather than to Algebra or Trigonometry.

New matter has been added in the treatment of taxation. Life, hail, and accident insurance are included in the chapter on insurance. In the treatment of stocks and bonds and exchange, the explanations are fuller than those usually given.

Sufficient elementary work is included to serve for whatever review may in any case be found advisable. The explanations are given in as simple language as possible and the methods illustrated are those that have been found by experience to be successful under modern conditions. The solutions are intended to lead the student in the process of thinking the problem out rather than as pattern solutions to be copied and followed. An attempt is made to satisfy as far as possible the needs of the teacher who has many and varied subjects to teach, and also of the student who may have to depend largely on his own resources and the text book.

The more difficult theorems are given, not as so much theory to be mastered, but mainly for reference purposes, as well as to avoid the necessity of asking students to take on trust any of their working formulae that can be established without too great difficulty.

The problems are carefully selected and based on reliable information. It is not intended that students work every problem in one reading of the course. These are graded so as to give ample exercise for the average student and also to give the student with a special mathematical bent, scope for his power of interpretation, and practice suited to his talents. Problems involving the metric units are mixed with those involving the ordinary units thus giving metric measurements a place of equality with the older measurements and a more apparent relation to practical experience A number of arithmetical problems arising in Physics and Chemistry are included as properly belonging to Arithmetic. Miscellaneous exercises are provided for review purposes, giving practice in dealing with problems without the suggestion given by their arrangement under various headings.

For helpful suggestions the authors wish to thank the special committee on Arithmetical Text Books appointed by the Alberta Department of Education, and Mr. J. E. Campbell, B.A., of the Central Collegiate Institute, Regina.

For full, prompt, and courteous replies to requests for information, the authors are indebted to the Comptroller of Currency, the Fruit Commissioner, and the Dominion Statistician at Ottawa.

THE AUTHORS.

Calgary, 21 April, 1925.

CONTENTS

CHAPTER I

BASIC CONVENTIONS AND OPERATIONS

A. Notation and Numeration, Definitions.

Notation is the systematic representation of numbers by means of symbols. The ancients used either letters of the alphabet, or pictures, to represent numbers. We still use for certain purposes the Roman Numerals, which furnish an example of an early notation. Computation with these symbols is extremely difficult. Our decimal system of writing numbers we owe to the Arabs, who in turn obtained it from the Hindus.

In Roman and other old notations each symbol has but one value. In the decimal system each symbol has its intrinsic value and also several other values due to its position. The digit 3 has for its intrinsic value $1+1+1$, but if followed by another digit as in 36 it means 3×10, or in 368 it means $3\times(10)^2$, or in 6.3 it means 3 times one tenth. Hence our system is a *positional* system based on the number 10. Our numbers are said to be written in the *scale of* **10**.

The number 489 is a short form for $4\ (10)^2+8\ (10)+9$. Were we to use a duodecimal instead of a decimal system, the number 489 would mean $4(12)^2+8\ (12)+9$.

The digit 1, when standing alone, having its intrinsic value, is often called *unity*.

Numeration is the systematic expression in words, of numbers expressed according to any system of notation. Usage in Great Britain differs from usage in Canada and most other countries in enumerating numbers containing 10 or more digits. In Canada a billion is a thousand

millions, a trillion is a thousand billions, etc., while in Great Britain a billion is a million millions and a trillion a million billions.

Anything that admits of increase or decrease is a *magnitude*. All numbers, lengths, volumes, weights, etc., are magnitudes.

An object, a group of objects, or a magnitude with which other objects or magnitudes of the same kind are compared for purposes of measurement, is called a *unit*.

A measured magnitude is called a *quantity*. 15, x, 9 ft., $2\frac{1}{2}$ hrs., are all quantities. 15 and x, consisting of numbers only, may be called *abstract quantities*. 9 ft., and $2\frac{1}{2}$ hrs., in each of which a number is attached to the name of a unit of which it indicates repetitions, part or parts, are *concrete quantities*.

While the simple rules apply primarily to numbers, they are extended freely to operations with concrete quantities.

A *factor* (or *measure*) of a number is any whole number that will divide into it without a remainder. 1, 2, 3, 4 and 6 are factors of 12. Unity is of course a factor of every whole number. A number that has no factors except itself and unity is a *prime number*. 1, 3, 5, 7, 11, 13, 17, etc., are prime numbers. Numbers exactly divisible by 2 are *even numbers*, and numbers not divisible by 2 are *odd numbers*. Unless otherwise specified the word "number" in this chapter means a whole number, that is, an *integer*.

A *multiple* of a number is a number that contains it as a factor. 12 is a multiple of 2, 3, 4 and 6, since 2, 3, 4 and 6 are all factors of 12. 12 is called a *composite* number as it has factors other than unity and itself.

B. Simple Rules, Short Methods.

The "borrowing" process in subtraction is often eliminated in practice by what has been called the "making change" method. A customer buys an article for \$2.85 and gives a \$5 bill in payment. The merchant begins with \$2.85 and *adds on* enough to bring this sum up to \$5 as he counts out the change, first 15 cents to make \$3, then \$2 more to make the \$5. The difference is thus obtained by addition. Similarly in the subtraction:

9426	Beginning in each case with the subtrahend,
3782	$2+4=6$, Put down the 4,
5644	$8+4=12$, Put down the 4, carry 1
	$8+6=14$, Put down the 6, carry 1
	$4+5=9$, Put down the 5.

The two sides of a ledger or cash book are similarly balanced by addition:

\$ 3.85	\$ 1.50
11.42	10.94
6.18	2.60
.80	*7.21*
\$22.25	\$22.25

Having added the left hand column, the sum \$22.25 is entered under both colums. Adding downward:

$4+\mathit{1}=5$, Put down 1

$5+9+6+\mathit{2}=22$, Put down 2, carry 2

$3+2+\mathit{7}=12$, Put down 7, carry 1

$1+1+\mathit{0}=2$,

The difference to be written in is \$7.21.

If a subtrahend is such a number as 9998, instead of by subtracting in the usual manner, the difference may be

found by subtracting 10000 and adding 2. $58327-9998=58327-(10000-2)=58327-10000+2=48329$.

Certain short cuts in multiplication are worth observing:

Since adding a zero to a whole number multiplies it by 10, adding a zero and dividing by 2 will multiply a number by 5. Similarly adding 2 zeros and dividing by 4 will multiply a number by 25. Adding 3 zeros and dividing by 8 will multiply a number by 125. Adding 2 zeros and dividing by 3 will multiply a number by $33\frac{1}{3}$.

Since 10 times a number, less the number is 9 times the number, adding a zero and subtracting the original number from the result will multiply a number by 9. Adding a zero and adding the original number to the result will multiply a number by 11.

To square a number ending in 5, square the 5, giving 25; multiply the part before the 5 by a number 1 greater than itself and put the product before the 25, thus:

To square 65,

$$5\times5=25$$
$$6\times7=42$$

The square is 4225.

Since $65=60+5$

$$\begin{aligned}(65)^2&=(60+5)^2\\&=60\times60+60\times5+5\times5\\&\qquad\qquad\;\;+60\times5\\&=60\times60+60\times10+5\times5\\&=60(60+10)+25\\&=4200+25=4225.\end{aligned}$$

The method will give the square of any number ending in 5.

In division, if the divisor may be expressed as the product of two or more simple numbers, the process of

long division can often be accomplished by performing two or more short divisions. It will be effort well repaid to learn the multiplication tables for 13, 17 and 19 up to 9 times each, both for multiplication and division, but especially for division.

To divide 49692 by 385.

$$385 = 5 \times 7 \times 11$$

$$\begin{array}{rl} 5\,/\underline{49692} & \\ 7\,/\underline{9938} & -2 \text{ units remain} \\ 11\,/\underline{1419} & -5 \text{ fives remain} \\ 129 & -0 \text{ thirty-fives remain} \end{array}$$

Quotient, 129

Remainder $2+25=27$.

If the division be exact the process is shortened somewhat further.

C. Tests for Divisibility

I. A number is divisible by 2 if its last digit is divisible by 2.

For example, $796=790+6=79\times 10+6$. Since 10 contains 2 as a factor, 79×10 contains 2 as a factor. The divisibility depends upon the last digit. Since 6 is divisible by 2, 796 is divisible by 2.

Note: Zero is divisible by all finite numbers without a remainder, that is, all numbers are factors of zero.

II. A number is divisible by 5 if its last digit is divisible by 5, that is, if the last digit is 5 or zero. The proof is simply a repetition of that for divisibility by 2.

III. A number is divisible by 4, if its last two digits form a number divisible by 4.

For example, the number $1376=1300+76=13\times100+76$. Since 100 is divisible by 4, 13×100 is divisible by 4, so that 1376 is divisible by 4 if 76 contains 4 as a factor.

IV. A number is divisible by 25 if the last two digits form a number divisible by 25. The proof is the same as for divisibility by 4.

V. A number is divisible by 8 if the last three digits form a number divisible by 8.

VI. A number is divisible by 125 if the last three digits form a number divisible by 125.

VII. A number is divisible by 3 if the sum of its digits is divisible by 3.

For example, the number 47235 may be broken up thus:

$$47235=\left.\begin{array}{r} 40000 \\ +\ 7000 \\ +\ \ 200 \\ +\ \ \ 30 \\ +\ \ \ \ 5 \end{array}\right\} \begin{array}{l} =4(9999+1) \\ \ \ +7(999+1) \\ \ \ +2(99+1) \\ \ \ +3(9+1) \\ \ \ +\ \ 5 \end{array}\left.\vphantom{\begin{array}{l} 1\\1\\1\\1\\1 \end{array}}\right\} \begin{array}{ll} =4\times9999 & +4 \\ \ \ +7\times999 & +7 \\ \ \ +2\times99 & +2 \\ \ \ +3\times9 & +3 \\ & +5 \end{array}$$

$=(4\times9999+7\times999+2\times99+3\times9)+(4+7+2+3+5)$.

Since 9999, 999, 99 and 9 all are divisible by 3, $4\times9999+7\times999+2\times99+3\times9$ is divisible by 3. The divisibility of 47235 by 3 then depends on the divisibility by 3, of $4+7+2+3+5$, that is, of the sum of its digits. It is evident that any number can be broken up in this way, and the same reasoning applies.

VIII. A number is divisible by 9 if the sum of its digits is divisible by 9. Substituting 9 for 3 the preceding investigation holds for 9 just as for 3.

IX. A number is divisible by 11 if the difference between the sum of the digits in the odd places and the sum of the digits in the even places is divisible by 11. The investigation is similar to that for 3 and 9.

Take for example 29163.

$$29163 = \left.\begin{array}{r} 20000 \\ +\ 9000 \\ +\ 100 \\ +\ 60 \\ +\ 3 \end{array}\right\} = \left.\begin{array}{r} 2(9999+1) \\ +9(1001-1) \\ +\ 1(99+1) \\ +\ 6(11-1) \\ +\ 3(1) \end{array}\right\} = \begin{array}{r} 2\times 9999+2 \\ +9\times 1001-9 \\ +\ 1\times\ 99+1 \\ +6\times 11-6 \\ +3 \end{array}$$

$=(2\times 9999+9\times 1001+1\times 99+6\times 11)+(3+1+2-9-6)$

$=(2\times 9999+9\times 1001+1\times 99+6\times 11)-(9+6-2-3-1)$

11, 99, 1001, 9999, are all multiples of 11.

The divisibility then depends on the difference between the sum $9+6$ and the sum $2+3+1$, that is, the difference between the sum of the digits in the odd places and the sum of the digits in the even places.

X. Divisibility by composite numbers like 6, 12, 15, 18, 20, 22, etc., can be established by testing for their factors. A number is divisible by 6 if the sum of its digits is divisible by 3 and the last digit is even. A number is divisible by 12 if the tests for 3 and 4 show both of these to be factors. For 15, apply the tests for 3 and 5. For 18, apply the tests for 9 and 2, etc.

D. Accuracy

Simple Checks

It is essential in Arithmetic that absolute accuracy be attained in the simplest as well as the more complex operations. This should be accomplished even at the sacrifice of speed. Once the habit of accuracy is formed, speed may be acquired to some purpose. The work should be constantly checked, step by step, and the time so spent is well worth while.

Addition may be checked by adding upward the first time and downward in the checking. Adding the separate columns and putting the sums down separately in the proper vertical lines and then adding these will also serve as a check.

Subtraction may be checked by adding the remainder to the subtrahend to produce the minuend.

Multiplication may be checked by using the multiplier as multiplicand and the multiplicand as multiplier, or by dividing the product by the multiplier to produce the multiplicand.

Division may be checked by multiplying the quotient by the divisor and adding in the remainder to produce the dividend. Often the divisor may be factored and the factors used as successive divisors. If the factors are small enough to be suitable for short division this is an effective check easily applied. Applying multiplication and subtraction checks to each step of the work may be preferable to either of the foregoing in some cases.

Casting out Nines

"Casting out Nines" is an accuracy check for multiplication, and it may also be used for division. While

it is not an absolute proof of correct work even when the check so indicates, it is an absolute proof of incorrect work when the check indicates an error. If the check shows no error the multiplication is correct in the great majority of cases.

It may be inferred from the test for divisibility by 9 that the remainder when a number is divided by nine is that obtained by dividing the sum of its digits by nine. When 6387141 is divided by 9 the remainder, if any, is obtained by dividing $6+3+8+7+1+4+1$ by nine. The numbers need not all be added together. Since $6+3=9$, and $8+1=9$, these four digits may be "cast out," or left out of the sum. The remainder is whatever is left over when $7+4+1$ is divided by 9. 12 divided by 9 leaves a remainder 3; when 6387141 is divided by 9 the remainder is 3.

By casting out nines it is found that 4684 leaves 4 as a remainder when divided by 9. Also 219 leaves 3 as a remainder when divided by 9.

$$\begin{array}{lrl}
\text{Dividing out} & 4684 = & 520\times 9+4 \\
 & 219 = & 24\times 9+3 \\
\hline
\text{The product} = & 520\times 24\times & 9\times 9 \\
 & + & 3\times 520\times 9 \\
 & + & 4\times 24\times 9 \\
 & + & 3\times 4 \\
\hline
\end{array}$$

Since 9 is a factor of the first three parts of this product, only 3×4 will give a remainder when the product is divided by 9.

When nines are cast out of the product of 4684 and 219 the remainder should be the same as when 3×4 is divided by 9. This constitutes the test of accuracy in the multiplication.

Example:

Multiply 38261 by 275.

38261	
275	
191305	
267827	
76522	
10721775	

In the multiplicand, $3+6=9$, $8+1=9$, remainder $=2$. In the multiplier, $7+2=9$, remainder $=5$. In the product the remainder should be the same as when 2×5 is divided by 9, that is, 1.

In the product, $7+2=9$, $7+1+1=9$. $7+5=12$. The remainder is 3. There is therefore an error in the work. It will be found that the third digit in the answer should be 5 instead of 7, which will make the remainder obtained from the product 1. The error was in addition. If necessary the several partial products may be checked by this method. It may be used to check addition and subtraction but is seldom used for these operations.

Reasonable Results

A further check, especially suitable for use in general problem work is obtained by making a rapid (generally mental) calculation of what the answer ought to be approximately. On comparing this with the result obtained by the longer process it may be found that this latter differs but slightly from the approximate result obtained and each calculation confirms the other. The difference may be so great however that the answer obtained may be rejected at once as unreasonable. It is well to sense the reasonableness of a result. These checks and approximations should be used before a problem is regarded as solved or any answer consulted.

Example:

Find the cost of shingling a roof 52′ 6″ long by 28′ 6″ wide at $7.75 a square (100 sq. ft.).

If this roof were 50′ by 30′ the area would be 1500 sq. ft., or 15 squares. At \$8 a square this would amount to \$120. A closer approximation would be \$7.75 × 15 or \$116.25. An answer like \$1000 or \$11.60 would be rejected as unreasonable on comparing it with either of these approximations.

Exercise Ia

(1) Show that the number 154,000,000 can be written in the following ways:

154×10^6; 1.54×10^8.

(2) Write the number 2.845×10^7 in our common notation.

(3) Obtain the quotients in the following without dividing:

$48375\div25$; $642825\div125$; $392625\div12\frac{1}{2}$;

$59400\div33\frac{1}{3}$; $49350\div16\frac{2}{3}$; $8375\div5$.

(4) Obtain the following products by division:

86324×25; 739425×125; $78436\times25\times125$;

$5763\times16\frac{2}{3}\times33\frac{1}{3}$.

(5) Divide 78492 by 315 using short divisions by factors of the divisor. Explain how to find the remainder.

(6) Using short methods, perform the following operations:

(a) Subtract

58496	32583	59362	483275
9998	992	9990	99980

(b) Multiply

3825×998; 693×990; 4927×499; 8276×9; 3694×11.

(7) One angle of a triangle is 87° 42′, another is 11° 51′. Find the third angle by addition.

Exercise Ib

(1) By using prime factors find the G. C. F. of:

(*a*) 180, 168 and 132.

(*b*) 14, 112 and 686.

(*c*) 125, 625, 600, 1200 and 275.

(2) Find the G. C. F. of:

(*a*) 12319 and 14605.

(*b*) 30592 and 60945.

(*c*) 315, 2267, 9012.

(*d*) 6893 and 6441.

(3) In the following express the L. C. M. as the product of the highest powers of the different prime factors:

(*a*) 36, 54, 63.

(*b*) 24, 27, 63, 68.

(*c*) 64, 144, 576, 512.

(4) Find the Greatest Common Factor of:

(*a*) 4 ft. 6 in., and 3 ft.

(*b*) 1 ft., 2 ft. 6 in., and 3 ft. 6 in.

(*c*) 24 pints and 7 gallons.

(5) Find the L. C. M. of:

(*a*) 81, 27, 324, 1584.

(*b*) 28, 63, 49, 798, 508.

(*c*) 36, 45, 72, 105, 150.

(6) Find the greatest number which when divided into 3912 and 92832 leaves remainders 40 and 25 respectively.

(7) If 1475 kilograms = 3245 pounds, what is the least number of kilograms equal to an exact number of pounds?

(8) Four bells begin tolling at the same time, and continue at intervals of 24, 30, 36, and 48 seconds respectively. How often will each bell have tolled when all again sound at the same instant?

(9) Find the greatest number which when divided into 75 and 109 leaves a remainder of 7 in each case.

(10) The circumferences of the wheels of a truck are 5 ft. 6 in., and 6 ft. respectively. If I start with the valves of a fore and a hind wheel at the bottom of the wheels, how far shall I have gone when next they are in the same position at the same moment?

Exercise Ic

(1) Devise a short method for each of the following and work mentally:

72 yds. cotton	@ $16\frac{2}{3}$¢	=	
96 lbs. butter	@ $33\frac{1}{3}$¢	=	
48 lbs. sugar	@ $6\frac{1}{4}$¢	=	
560 bu. oats	@ $37\frac{1}{2}$¢	=	
4800 bu. wheat	@ $87\frac{1}{2}$¢	=	
3200 bu. barley	@ $62\frac{1}{2}$¢	=	
1875 ft. lumber	@ \$40 per M.	=	
14500 shingles	@ \$5 per M.	=	
2750 lbs. hay	@ \$16 per T.	=	
4225 lbs. coal	@ \$8 per T.	=	
	$180 \times 44\frac{4}{9}$	=	
	$480 \times 56\frac{1}{4}$	=	
	$326 \times 62\frac{1}{2}$	=	
	$423 \times 33\frac{1}{3}$	=	
	$\$1.25 \times 196$	=	
	$1047 \times 66\frac{2}{3}$	=	

(2) Add the following and check the results.

(*a*)	(*b*)	(*c*)	(*d*)
4983	9765	7649	8127
7298	3987	8964	9872
5964	7893	9875	6898
8769	8238	6796	7685
9572	4586	5238	6276
7684	7372	4597	3897
6356	9853	3864	9761
7968	5487	9768	7426
3847	3879	6479	8969
4574	8438	4876	7892

(3) Find, mentally, the difference between 100 and each of the following numbers:

27, 18, 42, 35, 64, 56, 77, 82, 38, 88, 59.

(4) Beginning at the left hand digit write down the difference between:

279 and 1000, 3593 and 10000, 989,989 and 1,000,000, 437,280 and 1,000,000, 24,500 and 100,000.

(5) Subtract the following and check the work:

254,328	301,200	128,472	500,673
59,438	46,382	77,693	392,895

570,346	402,038	327,832	60,042
293,839	298,756	99,993	9,990

(6) Using short methods perform the following multiplications:

$$8759\times998;\quad 7296\times990;\quad 6874\times9995;$$
$$475\times147;\quad 8974\times246;\quad 68728\times168.$$

(7) Find, by casting out the nines, the remainder when each of the following numbers is divided by 9:

26,037,156; 452,367,120; 864,597,283; 3,769,285,428.

(8) Perform the following multiplications and check each result by casting out the nines:

328×486; 4687×365; 3925×563; 2876×748.

(9) Perform the following divisions and check each result by casting out the nines:

$7636\div92$; $6975\div75$; $27,456\div32$; $749,840\div86$.

(10) Monthly sales of a departmental store.

	Hardware	Clothing	Crockery	Shoes	Books and Stationery	
Jan.	$6,620.40	$37,338.60	$2,817.00	$11,258.38	$1,250.20	
Feb.	7,218.58	48,255.85	3,011.55	13,517.65	1,270.44	
Mar.	14,620.12	67,511.45	4,200.15	18,642.30	2,050.20	
Apr.	23,411.77	62,940.86	5,750.20	24,558.45	3,571.60	
May	22,016.05	48,417.93	2,619.38	16,611.27	2,433.86	
June	20,862.50	37,432.10	4,250.21	8,420.30	2,205.47	
July	18,483.21	24,316.95	1,640.11	7,860.43	1,508.67	
Aug.	15,721.58	12,827.42	1,433.15	6,672.10	820.50	
Sept.	7,460.91	48,852.49	3,290.40	18,003.80	1,540.92	
Oct.	10,430.72	65,419.87	4,372.54	25,680.15	2,453.16	
Nov.	8,444.68	71,611.36	4,681.16	30,572.63	3,674.28	
Dec.	6,320.18	126,892.24	10,823.50	35,438.26	8,038.16	

Find the total sales in all departments for each month and the total sales in each department for the year. Check by adding the two sets of totals.

CHAPTER II

COMPOUND QUANTITIES

For tables of weights and measures see page 285 and following pages.

The metric units have been obligatory in France since 1857. They are in use practically throughout the world except in the British Empire and the United States where they are used for all scientific investigations, but not generally used for ordinary weighing and measuring, although they may be legally used for all purposes.

The more complex units in common use did not even have fixed values up to comparatively recent times. The grain, meaning a kernel of wheat or barley, was the basis of the units of weight and length. The present unit of weight is the pound Avoirdupois, which has been fixed by Act of Parliament. A grain is 1/7000 of a pound Avoirdupois. The grain has the same value in Apothecaries' and Troy weights as in Avoirdupois. A pound Troy contains 5760 of these grains and a pound Apothecaries' the same. The ounce is 480 grains in Apothecaries' and Troy weights but an ounce Avoirdupois is $7000/16 = 437\frac{1}{2}$ grains. A carat is 1/150 of an ounce Troy or $3\frac{1}{5}$ grains. This weight is used in weighing precious stones. The same word is used in speaking of the fineness of gold. For example, gold 18 carats fine means gold with 18 parts by weight of pure gold out of 24, the balance of the 24 being alloy. The "long ton" is 2240 lbs., and the corresponding cwt. 112 lbs. These are still used for certain purposes.

The unit of length is the yard, which is now a fixed standard. A foot is a third part of a yard, and no longer subject to variation. An inch is a twelfth part of a foot. A mile is not now a thousand paces (mille passus) but 1760 yards. The rod and the furlong (furrow-long) have become fixed with the other units of length. The knot, or geographical mile, is 1 minute of the equatorial circumference of the earth, or about 6080 feet. The fathom, cubit, ell, hand, palm, digit, half days' journey, are other more or less ancient units of length. The fathom is still in use.

It is quite important that the relation between the common metric units and the English units be generally understood. The metre when instituted was supposed to be one ten-millionth of the distance from the equator to the pole on the earth's surface, but later measurements have shown that the calculation was not based on accurate data. The length of the metre however remains as originally fixed.

The metre is 39.37079 inches. 39.37 inches is close enough for most purposes. The decimetre is 3.937 inches, or nearly 4 inches. The centimetre is .3937 inches or about $\frac{2}{5}$ of an inch. The millimetre is .03937 inches or about a twenty-fifth of an inch. The kilometre is roughly $\frac{5}{8}$ of a mile. The litre is the contents of a cubic measure with 1 decimetre edge or about $\frac{9}{10}$ of a quart. The gram is the weight of 1 c.c. of distilled water at 4° C., or 15.43 grains. The kilogram is about 2.2 lbs. The stere (1 cubic metre) is used where we use the cubic yard or the cord. The tonneau, or metric ton, is about $\frac{1}{10}$ heavier than the ordinary ton, or a little lighter than the long ton. It is 2204.62 lbs.

In Great Britain pounds and decimals of a pound are used in most extended money calculations, the decimal

of a pound being reduced to shillings, etc., when it is necessary to express the result in terms of the smaller coinage.

In the metric system reductions are extremely simple. 1 Km. 8 Hm. 2 Dm. 5 m. 2 dm. 8 mm. = 182528 mm. = 1825.28 m. = 1.82528 Km. To bring mm. to dm. divide by 10, or move the decimal point one place to the left. To bring mm. to metres divide by 100, that is, move the decimal point two places to the left, etc. Conversely, to bring m. to mm. multiply by 100, or move the decimal point two places to the right.

In square measure each unit is 100 times the size of the next smaller unit and in cubic measure each unit is 1000 times the size of the next smaller unit.

Examples:

(1) To express £120. 15s. $8\frac{1}{4}$d. as pounds.

$8\frac{1}{4}$d. = 8.25d. Divide by 12 to bring to shillings.

8.25d. = .6875 shillings. Add the 15 shillings and divide by 20 to bring to pounds.

15.6875s. = .784375 pounds. Add 120 pounds, giving £120.784375, or £120.7844 to 4 places of decimals.

(2) To express £52.8745 in £. s. d.

To bring £.8745 to shillings multiply by 20, giving 17.49s.

To bring .49s. to pence multiply by 12, giving 5.88d.

Total £52. 17s. 5.88d., or £52. 17s. 6d., to the nearest farthing.

Exercises IIa and IIb provide a review of the compound rules, using the common units of measurement.

They also serve as further exercises in accuracy. IIc contains examples involving metric units.

Exercise IIa

(1) Reduce 2 ft. 8 in. to the fraction of a yard.

(2) Reduce 5 hr. 48 min. to the fraction of a day.

(3) A laborer worked 6 hr. 45 min. If a working day is 9 hours, what part of the day did he work?

(4) Reduce to the lowest denomination in each example:

(*a*) 10 rd. 4 yd. 2 ft. 8 in.

(*b*) 4 bu. 2 pk. 6 qt. 1 pt.

(*c*) 5 da. 18 hr. 27 min. 42 sec.

(*d*) 15 sq. rd. 24 sq. yd. 8 sq. ft. 96 sq. in.

(*e*) 2 T. 3 cwt. 24 lb. 12 oz:

(5) Reduce to higher denominations:

(*a*) 34372 pt. to bushels, pecks, etc.

(*b*) 3 qt. 1 pt. to a fraction of a peck.

(*c*) 30000 ft. to miles, yards.

(*d*) 743693 sec. to weeks, days, hours, etc.

(6) Change \$1200 to English money (£1 = \$4.86$\frac{2}{3}$).

(7) Change £1650. 12s. 6d. to Canadian money.

(8) Change \$683.60 to English money.

(9) How many pounds Troy are equal to 24 lbs. Avoirdupois?

(10) The area of a garden is 64 sq. rd. 4 sq. ft. What fraction of an acre is its area?

(11) A cubic foot of water weighs 1000 oz. Avoirdupois. Find the weight in grains of a cubic inch of water.

(12) What must be the length of a pile of stone 8 ft. high and 12 ft. wide to contain 21 cords?

Exercise IIb

(1) The lengths of the blackboards in a school room are 4 yd. 1 ft. 8 in., 3 yd. 9 in., 2 yd. 2 ft. 5 in., respectively. Find the total length of the three boards.

(2) From an iron rod 3 yd. 1 ft. 4 in. long, a blacksmith cuts off a piece 1 yd. 2 ft. 10 in. long. How much is left?

(3) In January a man had £25. 16s. 4d. in the savings bank. In February he deposited 18s. 6d., and in March a further sum of £1. 5s. 10d. How much had he then in the bank?

(4) The side of a square lot is 19 rd. 2 yd. 2 ft. 4 in. What is the perimeter?

(5) From 3 cwt. 27 lb. 4 oz. subtract 48 lb. 8 oz.

(6) Divide 29 mi. 2 yd. 2 ft. by 8.

(7) Divide 139 lb. 8 oz. by 4 lb. 8 oz.

(8) How much coal is there in three carloads of 38 T. 3 cwt. 41 lb., 29 T. 7 cwt. 5 lb., 32 T. 17 cwt. 70 lb.?

(9) How many steps does a man take in walking a mile if he advances 2 ft. 10 in., each step?

(10) How many times can a bushel measure be filled from a bin 8 ft. square and 6 ft. deep?

(11) How many gallons of water will a tank 4 ft. 7 in. by 2 ft. 11 in. by 1 ft. 3 in. contain?

(12) How many times is 7 ft. 6 in. contained in 195 mi. 280 rd.?

(13) A binder cuts a strip of 6 ft. each time it goes round a rectangular field. If the field is 40 rd. by 20 rd. find the area, in acres, of the grain left standing after the machine has been around 11 times.

(14) What is the rate in feet per second of a train which travels 30 miles per hour?

(15) If a cow gives 12 qt. 1 pt. of milk every day, and 1 lb. 8 oz. of butter can be made from 25 qt. of milk, how many pounds of butter can be made in one week from the milk of 20 cows?

(16) If each rail of a railroad weighs 90 lb. per yard, find, in tons, the weight of the rails required to lay 2 miles of railroad.

(17) A cubic foot of granite weighs 2600 oz. What is the weight of a cubic yard of granite expressed in tons and cwts., to the nearest cwt.?

(18) Find, to the nearest cent, the rent of 5 acres 120 sq. yds. of land at $15 per acre.

(19) Find the length of thread used in weaving a piece of silk 10 yd. long by 22 in. wide, if it is woven with 120 threads to the inch each way.

(20) A boy runs 100 yards in 12 seconds. If he could keep up this rate and he started to run a mile at 5 minutes past four, at what time would he finish?

Exercise IIc

(1) Express the following quantities, using all the units possible according to the metric system:

(*a*) 635,016 metres.	(*e*) 4325.5 grams.
(*b*) 4.96325 metres.	(*f*) 875,246 grams.
(*c*) 27085.2 metres.	(*g*) 264,591 litres.
(*d*) 96.45 grams.	(*h*) 2.0458 kilometres.

(2) (*a*) Express in approximate British measures the following:

20 cm., 8 m., $12\frac{1}{4}$ Km., 25 hectares, $12\frac{1}{2}$ litres, $10\frac{1}{4}$ kilograms.

(*b*) Express in metric system the following:

10 in., 50 ft., $12\frac{1}{2}$ mi., 5 acres, 3 pt., 500 gal., 14 lb., 4 oz.

(3) A motor car travels a measured 100 metres in 6 seconds. At how many kilometres per hour is it travelling?

(4) How many times can a piece 3 cm. 4 mm. long be cut from a rod 9 dm. long, and what length remains?

(5) Find, to the nearest $\frac{1}{10}$ inch, the inside diameter of the French 75 mm. field gun and the German 42 cm. howitzer.

(6) The area of a rectangular sheet of drawing paper is 35 sq. dm. If the width of the paper is 450 mm., find its length to the nearest mm.

(7) How many cubic metres of ice are there on a pond covering 20 Ha. if the ice averages 4 cm. in thickness?

(8) Add together the following:

(*a*) 34 m., 605 cm., 990 mm., reducing the result to Dm.

(*b*) 4 dm., .068 Km., 5 mm., reducing the result to cm.

(9) Light travels with a velocity of 3×10^{10}. cm. per second. Find, in kilometres, the distance of a star, given that light takes 50 years to traverse the distance.

(10) How many kilograms of water will a rectangular cistern 3.25 m. long, 2.5 m. wide and 2.75 m. deep, hold?

(11) A pipe is estimated to hold 3 litres per metre length. How many hectolitres would the pipe hold if it were 12 Km. long?

(12) A man measures a field with a decametre chain and makes the length 35 Dm. He afterwards found that his decametre chain was 2 dm. too short. What is the real length of the field in metres?

(13) A litre of oil weighs 875 g. Find (correct to the nearest litre) the number of hectolitres of oil that weigh 1000 Kg.

(14) A cyclist notices that he passes a hectometre post every 18 seconds. Calculate his speed in kilometres per hour.

(15) Assuming that water expands 10% in freezing find the weight, in kilograms, of one stere of ice.

(16) A motor car travels for the first 20 minutes at the rate of 6 Km. 126 m., per hour, then for 2 hrs. 20 minutes at the rate of 30 Km. per hour, and afterwards for 25 minutes at the rate of 7 Km. 152 m., per hour. Find the total distance travelled by the car.

(17) What is the weight of:

(*a*) 25 c.c. of gold (specific gravity = 19.3)?

(*b*) 15.52 litres of olive oil (specific gravity = .915)?

(*c*) 1580 cubic decimetres of ice (specific gravity = .917)?

(18) A metre is approximately 39.37 inches. Find the number of litres in a cubic foot.

(19) Find, to the nearest yard, the difference between 80 kilometres and 50 miles.

(20) A train takes 25 minutes to pass through a tunnel 10750 m. long. Find the speed of the train in miles per hour.

CHAPTER III

VULGAR FRACTIONS

A. Elementary Concepts

The word "fraction" denotes that the unit has been *broken* up. If unity be broken, or divided, into two equal parts, each part is called a half, if into three equal parts, each part is a third, if into twenty equal parts, each is a twentieth.

Instead of writing "one twentieth" or "1 twentieth," the shorter form $\frac{1}{20}$ is used. The 20, or *denominator*, indicates the number of parts into which unity is divided, thus determining the magnitude of each part and so serving as the name of one part. More than one of the equal parts of unity may be under consideration. In $\frac{3}{4}$, three parts are taken, each part being one of the 4 equal parts of unity. In $\frac{8}{5}$, unity is divided into 5 equal parts and 8 of such parts are specified. The 8 is called the *numerator*, as it indicates the number of parts taken.

$\frac{3}{4}$ is called a *proper* fraction. In it not all of the parts of the unit are taken. $\frac{8}{5}$ is called an *improper* fraction because more than all of the parts making up unity are taken, or specified. $\frac{5}{5}$ is considered an improper fraction because strictly speaking it is not a fraction at all, but a complete unit written in fractional form. An improper fraction such as $\frac{11}{4}$ may be expressed as the sum of a whole number and a proper fraction. $\frac{11}{4}=\frac{8}{4}+\frac{3}{4}=2+\frac{3}{4}$ or $2\frac{3}{4}$ which is known as a *mixed number*.

As we proceed the idea of the fraction is enlarged to include that of an indicated division. $\frac{11}{4}$ is equivalent to $11 \div 4$, which is $2\frac{3}{4}$, as already explained.

B. Comparison of Magnitudes. Addition and Subtraction

To compare two fractions as to magnitude bring them to equivalent fractions with a common denominator.

To compare $\frac{5}{7}$ and $\frac{9}{13}$. 91 is a multiple of 7 and of 13.

$$\frac{5}{7}=\frac{5\times 13}{7\times 13}=\frac{65}{91};\quad \frac{9}{13}=\frac{9\times 7}{13\times 7}=\frac{63}{91};$$

which shows $\frac{5}{7}$ to be the larger.

The fractions may be added when brought to the same denominator.

$$\frac{5}{7}+\frac{9}{13}=\frac{65}{91}+\frac{63}{91}=\frac{128}{91}.$$

Similarly, $\frac{5}{7}-\frac{9}{13}=\frac{65}{91}-\frac{63}{91}=\frac{2}{91}.$

More than two fractions can be combined in the same manner.

In $\frac{5}{8}+\frac{1}{6}+\frac{7}{12}+\frac{5}{9}$, 72 is the least number that will contain 8, 6, 12 and 9 evenly and hence the least common denominator of the fractions. Bring them all to fractions with denominator 72. The work may be abbreviated thus:

$$\frac{45+12+42+40}{72}=\frac{139}{72}=1\frac{67}{72}.$$

72 contains 8 nine times, 9 times 5 is 45. Put down 45. 72 contains 6 twelve times, 12 times 1 is 12. Put down 12, etc. In the same way,

$$\frac{5}{8}+\frac{1}{6}+\frac{7}{12}-\frac{5}{9}=\frac{45+12+42-40}{72}=\frac{59}{72}.$$

C. Multiplication and Division

The number 3 is obtained thus, $1+1+1$.

3 times 8 is obtained thus, $8+8+8$.

3 being a whole number, multiplying 8 by 3 is really finding the sum of 3 eights just as 3 is the sum of 3 units. If the multiplier is fractional, such as $\frac{1}{3}$, or $\frac{5}{8}$, multiplication involves something more than a series of additions.

The fraction $\frac{1}{3}$ is obtained by dividing unity into 3 equal parts, and taking one of such parts. $\frac{1}{3}$ times 12 is obtained by dividing 12 into 3 equal parts and taking 1 of such parts, giving 4. $\frac{2}{3}$ times 12 is found by dividing 12 into 3 equal parts and taking 2 of such parts. Hence, to multiply by a fraction perform the same operation on the multiplicand, as is performed on unity, to produce the fraction. This definition applies equally to multiplication by a whole number or a fraction.

To multiply $\frac{5}{7}$ by $\frac{2}{3}$, first divide $\frac{5}{7}$ into 3 equal parts. Each of those parts will be $\frac{1}{3}$ of $\frac{5}{7}$. Take 2 of such parts. Evidently this will give $\frac{2}{3}$ of $\frac{5}{7}$, that is, multiplication is reduced to the "of sign."

$$\frac{5}{7}\times\frac{2}{3}=\frac{2}{3}\text{ of }\frac{5}{7}=\frac{10}{21}.$$

To divide $\frac{5}{7}$ by $\frac{2}{3}$ is to find the fraction which when multiplied by $\frac{2}{3}$ will give $\frac{5}{7}$. In other words, when the required fraction is divided into 3 equal parts, 2 of the parts give $\frac{5}{7}$.

One part will be $\frac{1}{2}$ of $\frac{5}{7}$

The three parts will be $\frac{3}{2}$ of $\frac{5}{7}$ or $\frac{5}{7}\times\frac{3}{2}=\frac{15}{14}$.

Hence to divide by a fraction, invert the divisor and multiply.

If a symbol such as x be used for the required quotient in the division of $\frac{5}{7}$ by $\frac{2}{3}$, then $x \times \frac{2}{3} = \frac{5}{7}$

$$\frac{2}{3}x = \frac{5}{7}$$

$$\frac{1}{3}x = \frac{1}{2} \text{ of } \frac{5}{7}$$

$$x = \frac{3}{2} \text{ of } \frac{5}{7} = \frac{5}{7} \times \frac{3}{2} = \frac{15}{14}.$$

Sometimes a series of multiplications, divisions, additions, subtractions and "of signs" follow one another as in:

$$\frac{1}{2} + \frac{2}{3} \times \frac{1}{8} - \frac{1}{6} + \frac{3}{4} \div \frac{2}{5} \text{ of } \frac{3}{8}.$$

First eliminate the "of sign," next perform the multiplications and divisions, and finally, the additions and subtractions, thus:

$$\text{The above } = \frac{1}{2} + \frac{2}{3} \times \frac{1}{8} - \frac{1}{6} + \frac{3}{4} \div \frac{3}{20}.$$

$$= \frac{1}{2} + \frac{1}{12} - \frac{1}{6} + \frac{5}{1}$$

$$= \frac{6+1-2+60}{12} = \frac{65}{12} = 5\frac{5}{12}.$$

If for any reason additions or subtractions are required before multiplications or divisions the fractions to be combined by adding or subtracting should be enclosed in brackets, thus:

$$\left(\frac{1}{2} + \frac{2}{3}\right) \times \frac{5}{8} - \left(\frac{1}{6} + \frac{3}{4}\right) \div \frac{3}{2}.$$

$$= \frac{7}{6} \times \frac{5}{8} - \frac{11}{12} \times \frac{2}{3} = \frac{35}{48} - \frac{11}{18} = \frac{105-88}{144} = \frac{17}{144}.$$

Should the L. C. M. or G. C. M. of a number of fractions ever be required it may be stated that the L. C. M. of the fractions =

$$\frac{\text{the L.C.M. of the numerators}}{\text{the G.C.M. of the denominators,}}$$

and the G.C.M. =

$$\frac{\text{the G.C.M. of the numerators}}{\text{the L.C.M. of the denominators.}}$$

Exercise IIIa

(1) Simplify the following

(*a*) $(11+5\tfrac{3}{10})+2(\tfrac{3}{8}+2\tfrac{11}{20})-3(2-1\tfrac{4}{5})$

(*b*) $$\frac{\frac{3}{5}\left\{1-(\frac{2}{3})^4\right\}}{1-\frac{2}{3}}$$

(*c*) $$\frac{1}{2}-\frac{(\frac{1}{2})^3}{3}+\frac{(\frac{1}{2})^5}{5}$$

(*d*) $$\left\{\frac{\frac{7}{11}-\frac{2}{5}}{45-\frac{5}{11}} \text{ of } 7\frac{7}{13}\right\}+\left\{\frac{1\frac{1}{4}}{5\frac{3}{4}} \text{ of } 1\frac{1}{5}\right\}$$

(*e*) $$\left\{\frac{3\frac{1}{4}}{2\frac{1}{2}} \text{ of } 4\frac{1}{3}\right\}-\left\{2\tfrac{1}{2}\div 6\frac{1}{5}\right\}$$

(2) Find the value of:

$$\left\{1-(\frac{1}{4}+\frac{1}{5})\right\}+\left\{2-(\frac{2}{4}+\frac{2}{5})\right\}+\left\{3-(\frac{3}{4}+\frac{3}{5})\right\}$$

(3) $138\tfrac{1}{2}+13\tfrac{1}{2}=22\tfrac{3}{20}\ (k+4\tfrac{1}{2})$. Find the value of k.

(4) Simplify:

$$\frac{4\frac{7}{8}-8\frac{3}{5}+7\frac{2}{3}}{5\frac{3}{16}-6\frac{3}{10}+9\frac{5}{6}} \text{ of } \left\{\frac{23}{43}\div\frac{11}{13}\right\}.$$

(5) Find the simplest form of the fraction:

$$\frac{2\frac{1}{3}-1\frac{1}{2} \text{ of } 1\frac{1}{4}}{(2\frac{1}{3}-1\frac{1}{2})\times 1\frac{1}{4}} \div 1\frac{1}{5}.$$

(6) Find the value of $\sqrt{s(s-a)\ (s-b)\ (s-c)}$ when $a=21$, $b=28$, $c=35$, $s=\frac{a+b+c.}{2}$ (The square root should be found by means of prime factors.)

(7) What must be added to the sum of $\frac{11}{36}$ and $\frac{13}{48}$ so as to make it equal to unity?

(8) If $x-3\frac{1}{8}$ is equal to $2\frac{7}{10}$, what must be the value of x?

(9) Subtract the sum of 5 and $2\frac{11}{15}$ from $13\frac{19}{35}$.

(10) A sheet of tin is $\frac{8}{9}$ of a foot long and $1\frac{5}{16}$ inches wide. What is the total area of 18 such sheets?

(11) The edges of a rectangular solid are $3\frac{1}{5}$ in., $4\frac{5}{8}$ in., and $1\frac{7}{8}$ in. Find (1) the total length of its edges, (2) the total area of its faces, (3) its volume.

(12) Find the difference in volume between a cube whose edge is $3\frac{1}{4}$ in., and a rectangular solid which is $4\frac{2}{3}$ in. by $3\frac{1}{7}$ in. by $1\frac{3}{4}$ in.

(13) Find the value of $3x^2+5x-1\frac{1}{4}$ when $x=1\frac{3}{5}$.

(14) A litre is nearly equal to $1\frac{3}{4}$ pints. Find the number of pints in $7\frac{3}{10}$ litres.

(15) A fraction is increased by $\frac{1}{3}$ and then multiplied by 14, and the result is $16\frac{2}{3}$. What is the fraction?

(16) The area of a rectangle is $\frac{21}{40}$ sq. in., and one side is $\frac{7}{8}$ of an inch. What is the length of the other side?

(17) The cubical contents of a glass tube of uniform section is $1\frac{4}{5}$ cu. in. If the height of the tube is $2\frac{1}{4}$ in., what is the area of the cross section?

(18) A kilometre is approximately $\frac{5}{8}$ of a mile. How many kilometres are there in $7\frac{3}{4}$ miles?

(19) Of a certain magazine $\frac{1}{2}\frac{1}{3}$ consists of advertisements and $\frac{1}{8}$ of illustrations. What fraction is reading matter?

(20) A map is drawn to the scale of 12 miles to $\frac{1}{2}$ inch. What is the distance between two towns that are $7\frac{3}{4}$ inches apart on the map?

(21) Working $9\frac{1}{2}$ hours a day, a man can finish a piece of work in 6 days. What fraction of the whole work can he do in a day of 8 hours?

(22) A father's age is $2\frac{3}{4}$ times his son's age. What fraction is the son's age of the father's?

Exercise IIIb

(1) Simpilfy: $\dfrac{\frac{1}{6}-\frac{1}{7}}{\frac{1}{7}-\frac{1}{8}}$ of $\dfrac{\frac{1}{8}-\frac{1}{9}}{\frac{1}{9}-\frac{1}{10}}$ of $\dfrac{\frac{1}{10}-\frac{1}{11}}{\frac{1}{11}-\frac{1}{12}}$.

(2) Simplify: $\frac{91}{122}(4\frac{1}{8}\times 6\frac{2}{7}+\frac{3}{14})\div\{4\frac{1}{8}$ of $(6\frac{2}{7}+\frac{3}{14})\}$.

(3) Simplify:

(*a*) $(\frac{1}{3}+\frac{4}{7})\dfrac{5\frac{1}{16}}{3\frac{6}{7}+2\frac{1}{4}}$.

(*b*) $\frac{3}{7}+\dfrac{3\frac{7}{8}-3\frac{1}{4}}{2\frac{1}{3}\times 9}$.

(*c*) $\frac{10}{9}\times\dfrac{\frac{1}{5}-\frac{1}{8}}{\frac{1}{2}-\frac{15}{32}}$.

(*d*) $\dfrac{31\frac{1}{3}-22\frac{1}{5}}{11\frac{1}{5}-1\frac{1}{7}}\div 1\frac{49}{88}+2\frac{5}{12}$.

(*e*) $\dfrac{\frac{1}{3}(\frac{1}{2}+\frac{5}{6})+\frac{4}{7}\times 2\frac{1}{10}}{4\frac{3}{4}-\frac{2}{3}\times 5\frac{2}{5}}$.

(*f*) $\dfrac{2\frac{5}{12}}{\frac{5}{6}+\frac{3}{8}}\times\dfrac{\frac{5}{6}-\frac{3}{8}}{4\frac{3}{8}}$.

(4) A tourist loses his purse containing $\frac{3}{4}$ of his money, and decides to return home. His ticket costs $\frac{1}{3}$ of the money left. After buying his ticket he finds that he has $5 left. How much did he lose?

(5) A can run a mile in 7 minutes. B can run at the rate of 8 miles an hour. If they run at these rates a race of $3\frac{1}{2}$ miles, who will win, and by what distance?

(6) A tap can empty a bath in 11 minutes, and a second tap can empty it in 15 minutes. If both are open how long will they take to empty the bath?

(7) Some ink was spilled on a pile of blotting paper. It was estimated that each sheet of paper absorbed four fifths of the ink reaching it and let one-fifth through to the next sheet. Which sheet would be the first to receive less than $\frac{1}{1000}$ of the ink which was spilled?

(8) From an iron rod 5 ft. 11 in. long, how many bolts $3\frac{1}{2}$ in. long could be cut? What length will be left?

(9) A man bequeathed $\frac{5}{12}$ of his money to one son, $\frac{1}{2}$ of the remainder to another, and the balance to his widow. The difference between his sons' legacies was $1568. How much did the widow receive?

(10) To a gallon of a mixture consisting of five parts of milk and one part of water is added another quart of water. What fraction of the resulting mixture is milk?

(11) A merchant bought 4 bolts of cotton containing $40\frac{5}{8}$ yd., $30\frac{2}{3}$ yd., $52\frac{1}{4}$ yd., and $45\frac{3}{4}$ yd., respectively. He retailed $\frac{1}{2}$ of it at 50 cents a yard, $\frac{1}{3}$ of it at 45 cents a yard and the remainder at 32 cents a yard. What was the total selling price?

(12) An elastic ball is found to rebound to a height which is $\frac{2}{9}$ of that which it fell. On the third rebound it rises to a height of $\frac{4}{11}$ ft. From what height did it first fall?

(13) If I spend $\frac{5}{9}$ of my money what fraction of the remainder must I spend so as to have left $\frac{1}{6}$ of the whole?

(14) After paying $\frac{1}{15}$ of his income in taxes and $\frac{1}{12}$ of the remainder in insurance a man has left $1925. What is his income?

(15) The weight of an eight-day clock falls 4 feet in a week. In how many minutes does it fall $\frac{1}{4}$ of an inch?

(16) How far will a man walk between 9.15 a.m., and 10.20 a.m., if he can walk $1\frac{1}{4}$ miles in 25 miniutes?

(17) A water-cart which contained 360 gallons travels at the rate of $2\frac{1}{2}$ miles an hour, and is emptied after going $\frac{1}{4}$ mile. At what rate per second does the water flow out?

CHAPTER IV

DECIMAL FRACTIONS

A. Definitions, The Simple Rules

In the vast majority of problems connected with scientific investigations and industrial mathematics it is found that decimal fractions are much more satisfactory than vulgar fractions. They are more convenient because they can be manipulated just as whole numbers can be manipulated, avoiding such tedious work as reducing to common denominators before adding or subtracting.

Furthermore, all measures are *approximate.* They may be correct to the hundredth part of an inch, or the thousandth part of a centimetre, but there is a limit to their accuracy. These measurements can be expressed by decimals carried to the second decimal place, the third decimal place, or to any decimal place that will correspond to the degree of accuracy of the measurements themselves. To attempt to go beyond this in the calculations to be made is waste effort.

The decimal point in a number denotes no break in the law governing the positional values of the digits. Moving a digit one place to the left increases its value ten fold, and moving it one place to the right gives it a value one-tenth of its former value, whether the movement changes its place to the opposite side of the decimal point or simply affects its distance from the point. Consequently the presence of a decimal point in a number does not affect the application of the simple rules.

B. Contracted Multiplication and Division

Multiply 8.4291 by 12.03684, correct to 4 places of decimals.

8.4291		8.4291
12.03684		12.03684
3	37164	34
67	4328	674
505	746	5057
2528	73	25287
168582	0	1685820
84291		8429100
101.459728044		101.45972

Since the product is required to the fourth place only, the digits to the right of the vertical line simply determine what is to carry in the addition of the partial products. The shorter form gives as much of the work as is necessary for this. The multiplier 4 simply multiplies the 8. Looking one place to the right we see that $4 \times 4 = 16$. As this is nearer to 20 than to 10, carry 2 giving $8 \times 4 + 2 = 34$. The 8 multiplies only 84, the 6 only 842, etc.

The following arrangement, in which the digits of the multiplier are written in the reverse order, secures the partial products required and indicates where each multiplication is to begin.

```
  8.429100
  4 863021
----------
   8429100
   1685820
     25287
      5057
       674
        34
----------
 101.45972
```

The product is to be correct to *four* places. The partial products must be carried to *five* places. Place the *units* digit of the multiplier under the *fifth* decimal place in the multiplicand. Arrange the remaining digits of the multiplier so that all occur in the reverse order. Add two zeros to fill out the multiplicand. Multuply by 1, beginning with 0 directly above it. Multiply by 2 beginning with the zero directly above it. Multiply by 3 beginning with the 9. In multiplying by 6, there will be 5 to carry since $6 \times 9 = 54$. There will be 2 to carry in multuplying by 8 and also in multuplying by 4. In the sum mark off *five* decimal places. The result is 101.4597 correct to 4 places. If the digit following the fourth place be 5 or over, the fourth place digit should be increased by 1.

Divide 101.459728 by 12.03684 correct to 5 places of decimals.

Move the decimal place 5 places to the right in divisor and dividend. The divisor is now a whole number.

$12\cancel{0}\cancel{3}\cancel{6}\cancel{8}\cancel{4}$/10145972.8/8.42910

```
9629472
-------
 516500
 481474
 ------
  35026
  24074
  -----
  10952
  10832
  -----
    120
    120
    ---
      0
```

The divisor goes 8 times into the dividend. As this division includes the integral part of the dividend, the decimal point will follow the 8 in the quotient. In the

second division instead of bringing down the 8 in the dividend strike off the last digit of the divisor. Repeat this after each division, taking account of the number to carry in each multiplication.

It may happen that the division if performed in this manner does not give the required number of digits in the quotient before the divisor is reduced to 1 digit. In this case if the divisor is exact, one or more digits of the dividend must be brought down and part of the quotient obtained before beginning to strike digits off the divisor. If the divisor has several digits left after obtaining sufficient digits in the quotient, dividing both divisor and dividend by 100 or 1000 before the operation, using the nearest whole number as a divisor instead of the whole divisor, will shorten the process. A little practice along this line will be of more service than complicated rules. In most cases it will probably be as simple to begin with the complete divisor as to attempt to shorten further.

C. Changing Vulgar Fractions to Equivalent Decimal Fractions, Terminating and Circulating Decimals.

As a vulgar fraction may be considered an indicated division, to reduce a vulgar fraction to a decimal fraction, divide the numerator by the denominator and continue the division until there is no remainder, or until the digits of the quotient begin to recur in a definite order.

By division, $\frac{3}{4}$, $\frac{11}{50}$, $\frac{7}{125}$, reduce to .75, .22 and .056 respectively. The divisions are exact and the results are *terminating decimals.* $\frac{5}{6}$ reduces to .833333——— ad infinitum. This is a *repeating, circulating* or *recurring decimal.*

Any vulgar fraction the denominator of which can be made a power of 10 by multiplying by a whole number can be exactly expressed as a terminating decimal fraction.

$$\frac{3}{4}=\frac{3\times 25}{4\times 25}=\frac{75}{100}\text{ or }.75$$

$$\frac{11}{50}=\frac{11\times 2}{50\times 2}=\frac{22}{100}=.22$$

$$\frac{7}{125}=\frac{7\times 8}{125\times 8}=\frac{56}{1000}=.056$$

As 10 contains only the factors 2 and 5 any power of 10 is the product of a sufficient number of these factors. If any factor other than 2 or 5, the fraction being in its lowest terms, occurs in the denominator, the fraction cannot be expressed as a terminating decimal, because the denominator cannot be made a power of 10 by multiplying by any whole number.

$\frac{1}{3}$ or $\frac{5}{6}$ or $\frac{4}{7}$ therefore cannot be expressed as equivalent fractions with denominators powers of 10.

By division $\frac{1}{3}=.3333$ — — — ad infinitum, expressed thus: $.\dot{3}$

$\frac{5}{6}=.8333$ — — — ad infinitum, expressed thus: $.8\dot{3}$

$\frac{4}{7}=.5714285714285$, etc., ad infinitum, expressed thus: $.\dot{5}7142\dot{8}$

Any vulgar fraction that cannot be converted into a terminating decimal must give a repeating decimal. Take the case of $\frac{4}{7}$.

7/4.0/0.571428	
35	First division, quotient 0, remainder 4
50	
49	Second division, quotient 5, remainder 5
10	
7	Third division, quotient 7, remainder 1
30	Fourth division, quotient 1, remainder 3
28	
20	Fifth division, quotient 4, remainder 2
14	
60	Sixth division, quotient 2, remainder 6
56	
40	Seventh division, quotient 8, remainder 4

There can be only 6 remainders when dividing by 7 viz: 1, 2, 3, 4, 5, 6 in some order. When these have all occurred the same remainders must recur in the same order. The same digits will thus recur in the quotient in the same order giving a recurring decimal. In many cases the recurrence begins before all the possible remainders are used up. In dividing by 13 to reduce $\frac{5}{13}$ to a decimal only 6 of the possible 12 remainders occur. $\frac{5}{13} = .384615384 \ldots$

D. Changing Decimal to Equivalent Vulgar Fractions

$$.75 \text{ means } \frac{7}{10}+\frac{5}{100}=\frac{75}{100}$$

$$.823 \quad \text{"} \quad \frac{8}{10}+\frac{2}{100}+\frac{3}{1000}=\frac{823}{1000}$$

$$.0021 \quad \text{"} \quad \frac{2}{1000}+\frac{1}{10000}=\frac{21}{10000}$$

It is thus evident that a terminating decimal may be expressed as a vulgar fraction by writing for a denominator

1 followed by as many zeros as there are digits after the decimal point and using the digits of the decimal as a numerator.

Reduction of a circulating decimal to a vulgar fraction may be accomplished by multiplication of the decimal by 10 or some power of 10 such as will enable the recurring digits to be eliminated by subtraction.

(1) To reduce $.\dot{4}\dot{5}$ to a vulgar fraction

$$\begin{array}{lll} 100 \text{ times} & .\dot{4}\dot{5} = 45.454545 - & \text{ad infinitum} \\ \underline{1 \quad ``} & \underline{.\dot{4}\dot{5} = \phantom{45}.454545 -} & \underline{``\qquad ``} \\ 99 \quad `` & .\dot{4}\dot{5} = 45 & \\ & .\dot{4}\dot{5} = \frac{45}{99} = \frac{5}{11}. & \end{array}$$

(2) To reduce $.58\dot{6}$ to a vulgar fraction

$$\begin{array}{lll} 1000 \text{ times} & .58\dot{6} = 586.666666 - & \text{ad infinitum} \\ \underline{100 \quad ``} & \underline{.58\dot{6} = \phantom{5}58.666666 -} & \underline{``\qquad ``} \\ 900 \quad `` & .58\dot{6} = 528 & \\ & .58\dot{6} = \frac{528}{900} = \text{etc.} & \end{array}$$

From the above examples it may be inferred that a *pure circulating decimal*, that is, a decimal that has no non-repeating digits, may be expressed as a vulgar fraction by placing as a denominator a 9 for each digit in the repeating period, using the repeating period as a numerator. A *mixed circulating decimal*, that is, a decimal containing one or more digits that do not repeat, may be expressed as a vulgar fraction by placing as a denominator a 9 for each repeating digit in the repeating period, followed by a zero for each non-repeating digit following the decimal point. The numerator is the number formed

by the digits of the decimal to the end of the first repeating period, diminished by the number formed by the one or more non-repeating digits.

E. Circulating Decimals and The Simple Rules

Addition and subtraction of circulating decimals may be accomplished by reducing them to vulgar fractions before adding or subtracting and the sum or difference may be then expressed as a decimal. Or, by repeating the periods a sufficient number of times to produce a repeating period in the sum or difference the reduction to fractions may be avoided.

In the majority of cases absolute accuracy is not required. If the sum of the following numbers is required correct to 2 places of decimals the result will be 24.16, if to 3 places, 24.164, if to 4 places 24.1645.

$$\begin{array}{rrl}
4.5\dot{6}\dot{2} & = 4.56263 & \text{correct to 5 places} \\
15.\dot{3} & = 15.33333 & \text{correct to 5 places} \\
4.26\dot{8}5\dot{3} & = \underline{4.26854} & \text{correct to 5 places} \\
 & 24.16450 &
\end{array}$$

The addends are extended 1 place beyond the number of decimal places required in the result.

Subtraction may be performed in the same way.

In multiplication or division of circulating decimals it is necessary to reduce them to vulgar fractions before multiplying or dividing, if absolute accuracy is required. The product or quotient may then be expressed as a decimal if necessary. If the result is required correct to a certain number of decimal places contracted multiplication or division may be used and the reduction to vulgar fractions avoided.

Exercise IVa

(1) Write the following fractions in words: 3.03; .06; 4.89; .347; .008; 438.438; .0007; .0045; .000003; 4000.0004.

(2) Arrange the following decimal fractions in the order of magnitude, the greatest fraction being written first: .14; .0624; .989; .0119; .0598; .899; .9; .498; .5.

(3) Change the following decimal fractions to common fractions in their lowest terms:

.06; .65; .005; .00375; .9375; .12; .00025; .0425.

(4) Change the following common fractions to decimal fractions, accurate to four places of decimals:

$\frac{24}{25}$; $\frac{7}{9}$; $\frac{35}{40}$; $\frac{3}{125}$; $\frac{14}{898}$; $\frac{6}{7895}$; $\frac{835}{169}$; $\frac{1327}{625}$; $\frac{2}{3}$; $\frac{5}{11}$; $\frac{15}{37}$; $\frac{18}{55}$; $\frac{16}{27}$; $\frac{113}{288}$.

(5) Add the following numbers:

(a) .009, 346.98, 34.07, 1564.003, 4678.7, 8.9, 3.0048.

(b) 49.387, .892, 8.045, .0008, .00765, 148.6, 3584.03.

(6) Subtract as indicated:

(a) $2.0036-.78$; $67-.8896$; $57.4-9.6578$; $4.00758-.645$.

(b) $5-.0438$; $9.0003-.08763$; $38.6-29.763$; $1.05-.9863$.

(7) Change the following decimals into common fractions:

$.\dot{3}\dot{7}$; $.\dot{0}4\dot{5}$; $6.2\dot{1}1\dot{3}$; $.4\dot{2}\dot{5}$; $3.125\dot{4}$; $.13\dot{5}7\dot{8}$.

(8) Simplify the following correct to four places of decimals:

(a) $3.4\dot{7}+.\dot{0}4\dot{5}+13.\dot{4}$.

(b) $24.0\dot{2}\dot{8}-22.\dot{3}5\dot{7}$.

(c) $38+2.45\dot{9}-8.4\dot{3}2\dot{4}-.\dot{3}\dot{5}$.

(9) Find the sum of $\frac{5}{8}+\frac{3}{7}+\frac{9}{10}$ and express the sum as a decimal correct to three places of decimals. Expressing each fraction as a decimal to a sufficient number of places,

find the sum correct to three decimal places and compare results.

(10) The thickness of a book including the covers is found to be .038 metre; and the thickness measured without the covers is .029 metre. Find, in millimetres, the thickness of each cover.

Exercise IVb

(1) Write down the values of:

(1) 3.417×10^2. (2) $.04263 \times 10^3$. (3) $.328 \times 2 \times 10^2$. (4) $.00926 \times (200)^2$. (5) $2.078 \times 5 \times 10^4$.

(2) If one inch = 2.54 cm. (approximately) find to three decimal places the number of sq.cm. in a square inch. Find the area in sq.cm. of a sheet of paper which is 14.3 inches wide and 18.4 inches long.

(3) Find the value of $(.814 \times 2.42) - (.67)^2$.

(4) A boy has a box which is 27.4 inches long, 16.5 inches wide and 14.3 inches deep. He wishes to cover it (including top) with cloth which costs 45.5¢ per square yard. What is the least number of sq. ft. of cloth that he will require? What will the cloth cost him?

(5) The height of the mercury in a barometer is 75.8 cm. Find the height in inches, if one millimetre = .03937 inches.

(6) Write down the values of:

(1) $.408 \times 10^4$ (2) $5.87 \div 10^2$ (3) $36.497 \div 10^3$
(4) $9008 \div 10^5$ (5) $.645 \times 10^2 + 4.26 \div 10^2$.

(7) Find the value of $100 - \left\{\dfrac{1}{.4} + \dfrac{1}{(.4)^2} + \dfrac{1}{(.4)^3}\right\}$

(8) Divide the product of 61.83 and .117 by 2.43.

(9) Find the value of $1\frac{3}{4}+\frac{5}{8}-\frac{3}{16}+\dfrac{\frac{5}{16}}{2\frac{1}{2}}$ and express the result as a decimal.

(10) Find the area in square metres of the outside surface of a closed box 1.5 m. long, .09 metres wide, and 5.8 dm. high.

(11) Divide 4.812 by .0236 to 3 places of decimals. From your result write down the quotients of:

(1) $481.3 \div .236$ (2) $.4812 \div 2.36$.
(3) $4812 \div 236$. (4) $.04812 \div .00236$.

(12) Find the value of $\dfrac{.678 \times 9.01}{.0234}$ to three places.

(13) Evaluate $\dfrac{.744 \times 3.28}{(.615)^2}$ to three places.

(14) Find the product of $\frac{13}{24}$ and $\frac{19}{36}$ and express the result as a decimal correct to three places. Expressing each fraction as a decimal to a sufficient number of places, find the product correct to three places and compare results.

(15) Find, correct to five places of decimals, the value of the reciprocal of 3.204386.

(16) By utilizing your knowledge of factoring, find the value of

$$\frac{(.025)^3-(.0125)^3}{(0.25)^2-(.0125)^2}$$

(17) How many pieces 2.5 cm. long can be cut from a roll of wire 72.432 m. long? Express the remainder in mm.

(18) Find the value of $1.21 \times .121 \times .0121$ correct to four places.

(19) If a piece of cardboard 1.8 dm. long and 1.4 dm. wide weighs 50.4 grams, find the area, in square centimetres, of a piece which weighs 56.45 grams.

(20) A cubic decimetre of air weighs 1.293 grams, a cubic centimetre of water weighs 1 gram, and iron is 7.492 times as heavy as water. Find the volume of a piece of iron which has the same weight as 1000 cu. ft. of air.

Exercise IVc

(1) Find the sum of 2.4672, .28419, 34.635, .02186 correct to one place of decimals.

(2) Find the following sums:

(a) $26.769+.00273+165.295+1.6675$ to 3 places.
(b) $.02497+.003982+.14935$ to 3 places.
(c) $18.992+3.4221+.09997$ to the nearest integer.
(d) $\frac{1}{3}$ of $2.3+\frac{1}{7}$ of $12.5+\frac{1}{5}$ of $.0072$ to 2 places.

(3) Subtract as indicated:

(a) $8.2468-2.6524$ to 2 places.
(b) $6.53812-.79243$ to 3 places.
(c) $2.49384-1.79892$ to 3 places.

(4) Find the value of:

(a) $43.705\times .41634$ to 2 places.
(b) $.7138\times 27.96$ to one place.
(c) $.3482\times .9378$ to 3 places.
(d) 76.92×8.395 to the nearest integer
(e) 97.5648×24.0362 to 3 places.
(f) $.7825\times .09648$ to 4 places.
(g) 42.6532×27.4387 to 4 places.

(5) (a) Prove by contracted multiplication that $\sqrt{2}$ lies between 1.4142 and 1.4143 (multiply correct to 5 places).

(b) Also prove that the square root of 15 lies between 3.87298 and 3.87299 (multiply correct to 4 places).

(6) Perform the following divisions:

(a) $2.37145 \div 1.43619$ correct to 4 places of decimals.

(b) $5.6432 \div 7.47651$ correct to 4 places.

(c) $286.9 \div 461.514$ correct to 5 places.

(d) $2 \div 3.14159$ correct to 3 places.

(e) $74.615143 \div 42.869427$ correct to 2 places.

(f) $630408 \div 369842$ correct to 4 places.

(g) $.03684924 \div .4896378$ correct to 5 places.

(7) Find the cost of 6.755 metres of silver wire at 3.85 fr. per metre, to the nearest tenth of a franc.

(8) Divide \$1,562 among A, B, and C so that A may have .415 of the whole, B, .315, and C, the remainder.

(9) Find, to the nearest centimetre, the diameter of a circle which has a circumference of 115.8 decimetres. ($\pi = 3.1416$).

(10) Find the area of a rectangle 7.415 in. long and 4.256 in. wide, to the nearest $\frac{1}{10}$ of a square inch.

(11) The lunar month contains 29.53059 days; the year contains 365.2564 days. How many lunar months are there in a year (to the nearest integer)?

(12) The moon revolves round the earth in 27 days, 7 hrs., 43 min., 11.5 sec. How many revolutions does it make in 365 days (to hundredths of a revolution)?

(13) Find, to the nearest $\frac{1}{100}$ of an inch, the circumference of a circle which has a diameter of 8.97 in. ($\pi = 3.1416$).

(14) Find the area of a rectangle 5.425 m. wide and 10.246 m. long, to the nearest sq. dm.

CHAPTER V

THE ALGEBRAIC EQUATION, RATIO

A. Algebraic Solutions

The ability to use symbols to represent unknown numbers in the statement of the conditions of a problem in Arithmetic, and facility in the solution of the resulting equations, will be found to be of great service in problem work. It is quite legitimate to make full use of this algebraic knowledge in solving arithmetical problems. By means of the equation problems can be solved that would be much more difficult if not impossible of solution without it. Already in elementary Arithmetic the methods of Algebra have been used in a more or less disguised form.

Example: Divide $450 among A, B, and C so that B's share may be double A's and C's share three times B's.

Let A have 1 share, then B must get 2 shares and C, 6 shares. 9 shares = $450, 1 share = $50. A gets $50, B get $100, C gets $300. 1 share is an unknown quantity. If we let this unknown quantity be $x, then A gets $x, B gets $2x and C $6x.

$9x = 450$, $x = 50$, $2x = 100$, $6x = 300$. A gets $50, etc. The solutions are both algebraic.

In mensuration we have been using not only algebraic ideas but algebraic forms: $c = 2\ \pi r$ in the circle; $p^2 + b^2 = h^2$ in the right angled triangle. These are algebraic equations for which there is no convenient arithmetical substitute. Many problems in percentage may be simplified by the use of the equation.

Example: The price of a farm after being increased by 10% twice in succession is now \$8,470. What was the price before the increases?

Let $\$x$ be the original price.

This becomes $\frac{\$11}{10}x$ after the first increase.

and $\frac{11}{10}$ of $\frac{\$11}{10}x$ after the second.

$$\frac{121}{100}x = 8470$$

$$x = \frac{8470 \times 100}{121} = 7000$$

The original price was \$7,000.

This is more direct and simpler than supposing an original price and working the problem by a proportion.

B. Ratio and Proportion

The relation of one quantity to another quantity of the same kind, with regard to magnitude, is a *ratio.* The statement of the equality of two equal ratios is a *proportion.* The ratio of 3 to 7 may be written 3 : 7, or may be expressed in fractional form as $\frac{3}{7}$, since the magnitude of 3, the *antecedent,* to the magnitude of 7, the *consequent,* is determined by testing how many times, or parts of a time 3 will contain 7. If the ratio of a to b is the same as the ratio of 3 to 7, then, a : b :: 3 : 7. This latter statement is a proportion, reading, a is to b as 3 is to 7. It may be written $\frac{\mathrm{a}}{\mathrm{b}} = \frac{3}{7}$. The first and last terms of a proportion are called the *extremes,* and the second and third terms, the *means.* In $\frac{a}{b} = \frac{3}{7}$, a and 7 are extremes, b and 3 are means.

If $\frac{a}{b}=\frac{c}{d}$, multiplying by the common denominator, $ad=bc$, that is, the product of the extremes is equal to the product of the means.

Examples:

(1) If 8 articles cost \$44, what will 15 articles of the same kind cost?

Suppose 15 articles cost \$x. Evidently the first cost will have the same ratio to the second cost as the number of articles in the first case has to the number of articles in the second case.

Then $\frac{44}{x}=\frac{8}{15}$

$8x=44\times15,\ x=82\frac{1}{2}$.

15 articles cost \$82.50.

This method of solving problems by proportion has long been known as the "Rule of Three".

(2) Divide \$375 into 4 parts proportional to 6, 8, 7 and 4.

Let the first part be \$$6x$, then the second part is \$$8x$, the third part is \$$7x$, and the fourth part is \$$4x$.

$$6x+8x+7x+4x=375$$
$$25x=375$$
$$x=15.$$
$$6x=90,\ 8x=120,\ 7x=105,\ 4x=60$$

The parts are \$90, \$120, \$105 and \$60.

The volume of a gas decreases in the same ratio as the pressure to which it is subjected increases. That is, the volume is inversely proportional to the pressure.

Example:

If a given quantity of gas occupies 5 litres when the pressure is 760 mm. of mercury, what pressure will reduce it to 3 litres?

Since reduced volume is to original volume in the ratio 3 to 5.

Increased pressure is to original pressure in the ratio 5 to 3.

Required pressure $= \frac{5}{3}$ of 760 mm. $= 1266\frac{2}{3}$ mm. of mercury.

Plans of grounds, plans and elevations of buildings, bridges, or other structures, are examples of ratio or proportion. Each linear dimension of a plan bears a constant ratio to the corresponding dimension of what it represents. To secure this, plans are drawn to scale, a quarter inch, an inch, or other length on the plan, representing a greater length, such as a foot, or 10 feet, on the grounds or structure. Maps of territory are constructed in the same way.

Work problems may generally be solved by proportion. The work done in a given time is proportional to the number of workers, or the work done by a given number of workers is proportional to the time they work. The number of workers is proportional to the work to be accomplished in a given time and inversely proportional to the time allowed to complete a definite amount of work.

Examples:

(1) What is the height in feet of a tower that appears to be 15 in. high on an elevation drawn to the scale of $\frac{1}{8}''$ to the foot?

$\frac{1}{8}$ in. on plan represents 1 ft. on tower.

15 in. on plan represents $\dfrac{1}{\frac{1}{8}} \times 15$ ft. $= 120$ ft. on the tower.

The tower is 120 ft. high.

(2) If 40 men can complete a piece of work in 5 days of 9 hours each, how many men can complete twice as much work in 6 days of 8 hours each?

In 45 hours the work can be done by 40 men

In 45 hours twice the work can be done by (40×2) men

In 48 hours twice the work can be done by $\frac{40 \times 2 \times 45}{48}$ men $= 75$ men.

Labor may be saved in many problems by finishing the reasoning before taking time to work out the results of the separate parts. The expression giving the final result can then be simplified by cancellation.

Exercise Va

(1) Three man have their salaries, which are $2400, $2700 and $3600 per annum, increased in the same ratio. If the lowest salary be increased to $2800, to what will the others be increased?

(2) The mount of a picture is 16 in. by 9 in. It is desired to put it in a frame whose sides shall be in the same ratio as the sides of the picture. If the longer side of the frame is to be 20 inches, how long will the other side be?

(3) The area of a rectangle is 700 sq. in., and its sides are in the ratio of 4 : 7. What are the lengths of the sides?

(4) In a certain right angled triangle the ratio of the shortest side to the hypotenuse is 5 : 13. Find the ratio of the two sides containing the right angle.

(5) In what ratio must 75 be increased so that the result is equal to 90? In what ratio must it be decreased to reduce it to 60?

(6) The ratio of two numbers is 5 : 9. Their product is 1620. Find the numbers.

(7) Express each of the following ratios in its simplest form:

$\frac{1}{2}:\frac{1}{8}$; .9: 1; .75: 2; 3 yd.: 2 ft.; 3 sq. ft. : 4 sq. yd.; 1 lb.: 24 oz.; 3 T. 5 cwt.: 2 T.; $(135)^2 : (45)^2$.

(8) The ratio of the weights of equal volumes of ice and water is 0.917: 1. One cubic foot of water weighs 62.5 lbs. Find the difference in the weights, to the nearest pound, of a cubic yard of ice and of water.

(9) For Association football matches the maximum length and breadth of the playing field are 120 yd. and 80 yd. respectively. If on a certain ground the length taken is 110 yd., what should be its breadth so that the same ratio between the length and breadth may be preserved?

(10) A farmer divides his farm into arable and pasture land in the ratio of 3: 8. If the area of his arable land is 50 acres, what is the total area of his farm?

(11) Pure water is composed of two gases, oxygen and hydrogen, in the proportion by weight of 88.9 to 11.1. What weight of each is there in a cubic foot of water?

(12) Divide $200 among A, B and C so that A's share is to B's share as 2:1, and $16\frac{2}{3}$% of A's share is equal to C's share.

(13) A grocer mixed three kinds of tea worth 25¢, 30¢ and 35¢ a pound, in the proportion of 4: 3: 2, and sold the mixture at 40¢ a pound, thereby gaining $5.00. What quantity of each kind of tea was sold?

(14) A stick is broken into two pieces in the ratio of 2: 3. The smaller piece is again broken in the ratio of 4: 5. Find the ratio that each of the three pieces bears to the whole.

(15) The sides of four squares are proportional to the

numbers 1, 2, 3, 4 and the total area is 24.3 sq. in. Find the area of each square.

(16) Find the specific gravity, correct to one decimal place, of a substance of which 1 cu. in. weighs 1.4 oz.

(17) When a vertical rod 6 ft. high casts a shadow 9 ft. long, a tree casts a shadow of 150 ft. How high is the tree?

(18) Two boys are carrying a weight of 150 pounds, suspended upon a 9 ft. pole between them. The weight is 6 ft. from one boy and 3 ft. from the other. Neglecting the weight of the pole, how much does each boy carry if the loads borne are inversely proportional to the distances of the bearers from the weight?

(19) The pressure of compressed air varies inversely as its volume. If the pressure on the inner surface of a cylinder fitted with a piston be 20 pounds on the square inch, and when the piston is forced in 2 inches, the pressure becomes 30 pounds on the square inch, what is the length of the cylinder?

(20) A cwt. of tea is to be packed in three boxes whose capacities are in the ratio of 1.5: 2.5: 3. What weight will be packed in each box?

(21) The volume of a gas is directly proportional to the absolute temperature, and inversely proportional to the pressure. The volume of a certain quantity of gas at a temperature of 16°C. and pressure 758 mm., was 450 c.c. What will be its volume when the temperature is 25°C. and the pressure 770 mm.? (To find the absolute temperature add 273 to the Centigrade temperature).

(22) In the friction caused by a wheel rolling over a plane surface, it is found that the frictional resistance is approximately directly proportional to the load and

inversely proportional to the diameter of the wheel. In a certain instance the frictional resistance was equivalent to 850 lb. What will it be when the load is increased in the ratio of 5 : 3 and the diameter of the wheel decreased in the ratio of 5 : 6?

(23) The volumes of two blocks of ice are in the ratio of 5 : 6. When melted they produce $22\frac{1}{2}$ gal. of water. If in the process of melting the volume decreases by $\frac{1}{11}$ what were the volumes of the two blocks in cubic feet?

Exercise Vb

(1) A map is drawn to the scale of 1 in. to a mile. A park appears on this map as a rectangle 1.25 in. long and .5 in. wide. How many acres does the park contain?

(2) A picture measures 12 in. by 8 in. The picture is photographed and the short side of the photograph is 6 in. How long is the other side?

(3) The map of a country is drawn to the scale of .1 in to the mile. What area on the map will represent a lake covering 4000 acres?

(4) The distance between two towns, which are known to be 90 miles apart, measures 7.5 inches on a map. Find the scale of the map.

(5) A map is drawn to the scale of 5 in. to the mile. How many acres are there in a field which is represented on the map by a square whose side is 1.2 in.?

(6) The distance between two towns, which are known to be 45 miles apart, measures 3.75 inches on a map. Find the scale of the map (in miles to an inch).

(7) If we take a globe of one inch diameter to represent the earth, then we must have a globe of 9 feet in diameter at a distance of 323 yards to represent the sun

on the same scale. Taking the diameter of the earth as 7920 miles, find (a) the distance between the earth and the sun, (b) the diameter of the sun.

(8) 4 men can earn $180 in 10 days.

(a) In how many days will 12 men earn $120?
(b) How many men will earn $288 in 16 days?
(c) How much will 18 men earn in 6 days?

(9) 12 men working 8 hours a day earn $432 in 6 days.

(a) What will 8 men earn in 9 days of 10 hours each?

(b) How many hours a day must 16 men work to earn $2160 in 20 days?

(c) In how many days would 9 men earn $1215 working 12 hours a day?

(10) A besieged town has provisions for 100 days. After 20 days one-third of the inhabitants escape and the remainder are put on half rations. How long will the provisions last?

(11) If 10 men can do a piece of work in 6 days, what number of men would do 3 times the work in one-fifth of the time?

(12) Two apartment houses are built, one in 4 months and the other in 7 months. The number of workmen engaged on the first is double that employed on the second, and they work two hours a day overtime for which they are paid half as much again as for work done in a working day of 10 hours. If the sum paid for labor is $17,400, find how much of this sum was spent on each house.

(13) If a car goes $3\frac{1}{2}$ miles in 14 minutes, how far does it go in 25 minutes? How long will it take to go $19\frac{1}{4}$ miles?

(14) If a train takes $3\frac{1}{2}$ hours to complete a certain journey at 45 miles per hour, what time will it take at 42 miles per hour? At what rate must it travel to complete the journey in $4\frac{1}{2}$ hours?

(15) If 100 miners working 9 hours a day mine 5,000 tons of coal in 5 days, how many miners working 8 hours a day are needed to mine 9600 tons in 6 days?

(16) A garrison of 600 men has supplies for 40 days. How long will the supplies last if the garrison (1) is reduced to 500 men, (2) is increased to 800 men?

(17) If, by travelling $7\frac{1}{2}$ hours a day, I cover 375 miles in 10 days, how many hours a day must I travel in order to cover 210 miles in 9 days?

(18) A boy can do half as much work as a man. Two men and a boy can dig $2\frac{1}{2}$ yards of a certain ditch in $2\frac{1}{2}$ days. How long would it take 25 men to dig 25 yards of the same ditch?

(19) A contractor undertakes to finish a piece of work in 30 days and employs 16 men to do it. After 12 days only one-quarter of the work is done. How many additional men must be engaged in order to fulfil the contract?

(20) 12 bricklayers are set to build a wall, and the job is expected to take a week. When $\frac{1}{3}$ of the work is finished orders are given that the work must be completed in two days more. How many more men must be taken on?

(21) A contractor undertakes to complete a railway grade of $37\frac{4}{5}$ miles in $7\frac{1}{2}$ months. He employs 64 teams, but in $1\frac{1}{2}$ months has finished only $5\frac{3}{5}$ miles. Assuming the grade to be of uniform difficulty, how many additional teams must he now employ to complete the contract on time?

(22) A builder contracted to build a house in 12 weeks and agreed to pay \$25 per day or portion of a day that the work remained incomplete after the end of that period. To fulfil his contract he engaged 15 men to work 10 hours a day. After three weeks 10 of the men struck and remained away from their work for a fortnight. On their return the number of working hours was reduced to 9. What was due from the builder for his breach of contract?

(23) A contractor employs 15 men, each working 8 hours a day to complete a certain piece of work in 19 days. At the end of 10 days the work had to be suspended for two days owing to an accident in which 4 of the men were disabled. How many more men must he engage to complete the work in the specified time, all the men now working 9 hours a day?

CHAPTER VI

LONGITUDE AND TIME

The meridians run from pole to pole cutting the equator at right angles. The meridian line running through the Royal Observatory at Greenwich, England, is known as the *Prime* meridian. In travelling due east or west from Greenwich around the earth and back to Greenwich, a complete circle is described containing 360°. The meridian on the other side of the earth directly opposite the Greenwich or *prime* meridian is 180° from Greenwich, either way. All points on this meridian are described as 180° west longitude or 180° east longitude, or simply 180°. Points on the earth's surface west of the Greenwich meridian are in west longitude until 180° is reached. Similarly points east of the Greenwich meridian are in east longitude.

The earth revolves from west to east making a complete revolution in 24 hours. It turns 360° in 24 hours. Then it turns 1° in $\frac{24}{360}(=\frac{1}{15})$ of an hour = 4 minutes. Or putting it another way:

24 hours elapse while the earth turns 360°

1 hour elapses while the earth turns 15 degrees.

These two forms of the relation between longitude and time should be kept clearly in mind. 15 degrees correspond to 1 hour, and 1 degree corresponds to 4 minutes.

When it is noon at Greenwich on July 1st the sun is directly over the prime meridian. Points west of this meridian are turning towards the sun, that is, the sun is apparently approaching them and rising higher in the

sky or approaching sunrise according to the distance from the prime meridian. It is July 1st on this half of the globe, early in the morning, or in the forenoon. Points east of the prime meridian are moving away from the sun. It is afternoon or the first part of the night of July 1st. On the 180th meridian it is midnight. On one side of this line July 1st is just over; July 2nd is beginning. On the other side of the line July 1st is just beginning, or it is midnight of June 30th.

Twelve hours later it is noon on the 180th meridian. On its western side (east longitude) it is noon of July 2nd. On its eastern side (west longitude) it is noon of July 1st. This meridian with slight variations to suit local conditions is called the international date line. A ship going eastward crossing this puts its reckoning back from July 2nd to July 1st. Crossing it in the opposite direction it puts its reckoning on from July 1st to July 2nd,—or makes corresponding changes on any other date on which it crosses this meridian.

Examples:

(1) If it is 11 a.m. at Bridgeport, C.B., what time would it be at a point in the Rocky Mountains 120° west longitude, if Bridgeport is 60° west longitude.

The difference in longitude is 60°.

Then the difference in time must be 4 hours.

It is 7 a.m. at the point mentioned.

(2) If it is 6 a.m. Oct. 5th at Pekin (116° 26′ East), what is the true time at San Francisco (122° 26′ 15″ West)?

Difference in longitude is 238° 52′ 15″

15 degrees make a difference of 1 hour

238 degrees make a difference of 15 hrs., 52 min.

60′ longitude make a difference of 4 minutes time.
$52\frac{1}{4}'$ longitude make a difference of $3\frac{29}{60}$ minutes time, 3 min. 29 sec.

The total difference is 15 hrs. 55 min. 29 sec. It will not be 6 a.m. at San Francisco until this period of time elapses, hence it is 4 min. 31 sec. after two p.m., Oct. 4th.

Standard Time

If all places used true time, every movement east or west on the earth's surface would necessitate a change in time. A degree of longitude at the latitude of Edmonton would be roughly 40 miles. Then every 40 miles east or west of Edmonton would mean a change of 4 minutes, or about a minute to every 10 miles. To avoid confusion resulting from such a system, belts 15° wide adopt a uniform time throughout the belt, and the time in the next belt differs by 1 hour from this. A time belt is theoretically the belt extending $7\frac{1}{2}$° on each side of a time meridian. The time meridians are the prime meridian and 15°, 30°, 45°, 60°, 75°, etc., east and west. In practice the boundaries between time belts are irregular, suiting the convenience of our railway systems. Time reckoned in this way is called *standard time* and is used over the greater part of the world. The greater part of British Columbia uses the time belt of the 120th meridian. The time of this belt is called *Pacific time.* Alberta and Saskatchewan use that of the 105th meridian, *Mountain time*; Manitoba and western New Ontario, that of the 90th meridian, *Central time*; Eastern Ontario and Quebec that of the 75th meridian. *Eastern time.* The greater part of the Maritime Provinces and Newfoundland uses that of the 60th meridian, *Atlantic or Intercolonial time.*

Example:

What is the difference between true and standard time at Halifax, 63° 36′ W.?

Standard time for Halifax is the true time for 60° W. The difference is 3° 36′.

1° makes a difference of 4 min.

3° make a difference of 12 min.

36′ make a difference of $\frac{36}{60}$ of 4 min. = 2 min. 24 sec.

Required difference is 14 minutes 24 seconds.

Exercise VI

(1) What is the difference in longitude between two cities if the difference in time is 3 hr. 30 min. (true time)?

(2) If I start at Winnipeg and travel until my watch is 1 hr. 30 min., too fast, in what direction and how far do I travel (true time)?

(3) The first shock of the San Francisco earthquake was recorded at 5.12 a.m., at an observatory in 120° W., and at 19 min. 20 sec., past 8 a.m., at one in 75° W. If San Francisco and the two observatories are approximately in a straight line, what length of time was consumed by the shock in passing between the two observatories?

(4) The difference in time between Berlin, Germany, and San Francisco is 9 hr. 4 min. 18 sec. The longitude of San Francisco is 122° 26′ 15″ W. Find the longitude of Berlin (true time).

(5) The longitude of Paris, France, is 2° 20′ E. and the longitude of New York is 74° 3′ W. What time is it in New York when it is 7 p.m. in Paris (true time)?

(6) In travelling on a parallel of latitude from 78° 32′ 40″ West longitude, I notice that my watch has lost

1 hr. 20 min. 30 sec. In what direction am I travelling and what is the longitude of the second place (true time)?

(7) The longitude of Quebec is 71° 13′ 20″ W., and of Vienna 16° 20′ 22″ E. What is the time in Vienna when it is 9 a.m., in Quebec (true time)?

(8) What is the time at Calgary (114° West longitude) when it is 10 a.m. at Toronto, 79° 24′ West longitude? (Calgary uses Mountain Time).

(9) A town whose longitude is 117° 22′ 30″ W., uses standard time of 120° W. What is the correct standard time in the town when it is exactly noon by solar time?

(10) A ship at 54° 17′ 30″ W., sent a wireless message to one at 72° 24′ 50″ W., at 10 a.m. If transmitted without loss of time, when was the message received (true time)?

(11) At 8.40 a.m., a ship in longitude 17° 20′ W., sent a wireless message which was received by another ship at 8.45 a.m. Find the longitude of the second ship (true time).

(12) A cablegram, sent from New York in longitude 74° 0′ 3″ W., was received in Paris, France, in longitude 2° 20′ 22½″ E., at 1.30 p.m., after a delay of 25 minutes in transmission. At what time was it sent from New York (true time)?

(13) The longitude of Constantinople is 28° 59′ 15″ E. and that of Edmonton is 113° 28′ 45″ W. When it is noon by the sun at Edmonton, what is the solar time at Constantinople?

(14) The difference between the solar times of Calgary and Toronto is 2 hr. 18 min. 30 sec. If the longitude of Calgary is 114° 1′ 48″ W., what is the longitude of Toronto?

(15) At what places on a time belt is standard time true time? Where does standard time differ most from true time? Theoretically what is the maximum difference? On what part of the time belt is standard time ahead of true time? Behind true time?

(16) If it is 6 hr. 3 min. 12 sec., a.m. true time at Fort William when it is noon at Greenwich, what is the longitude of Fort William? Which is Fort William's time meridian? What is standard time at Fort William when it is noon at Greenwich?

(17) If the longitude of Vancouver is 125° 30′ W., how much are the Vancouver clocks ahead of true time when daylight saving is in force? (1 hour advance on standard time).

(18) A man's watch shows correct standard time when he leaves Vancouver for Halifax. What change will he have made in the time shown by his watch when he sets it by standard time at Halifax?

(19) What is the difference between standard time at Melbourne, 144° 58′ E., and standard time at Victoria, 124° 22′ W.?

(20) A ship calls at Rio Janeiro at 10 a.m. standard time. The ship's chronometer shows Greenwich time to be 1 p.m. True time on board ship is 7 min. 24 sec. ahead of standard time in the city. What is the longitude of Rio Janeiro?

CHAPTER VII

POWERS AND ROOTS

A. Powers of Numbers, Square Roots

When a number is multiplied by itself, that is, taken twice as a factor, the result is the square of the number. When taken as a factor three times the cube is obtained; when taken as a factor 6 times the sixth power is obtained, etc. The number of times any number is to be taken as a factor is indicated by a small digit, or digits, written above and to the right, thus:

5^2 is an abbreviation for 5×5, 4^3 is an abbreviation for $4\times4\times4$. The 2 and the 3 are called exponents or indices (sing. index).

5^2 $(=25)$ is the square or second power of 5, and 4^3 $(=64)$ is the cube or third power of 4.

The square of a fraction is obtained by squaring both numerator and denominator to produce the numerator and denominator of the square.

$$\left(\frac{3}{4}\right)^2=\frac{3}{4}\times\frac{3}{4}=\frac{3^2}{4^2}=\frac{9}{16}$$

$$\left(\frac{5}{6}\right)^3=\frac{5^3}{6^3}=\frac{125}{216}$$

The *square root* of a number is one of its two equal factors, that is, the number which when multiplied by itself will produce the given number. $4\times4=16$. Then 4 is the square root of 16, being one of its two equal factors.

The square root of a number which is a perfect square and easily factored may be readily obtained by reducing it to its prime factors or, often by expressing it as a product of square factors labor may be saved.

$$\begin{aligned} 32400 &= 2\times2\times2\times2\times3\times3\times3\times3\times5\times5 \\ &= 2^2\times2^2\times3^2\times3^2\times5^2 = (2\times2\times3\times3\times5)^2 \\ &= (180)^2. \text{ Then the square root of 32400 is 180.} \end{aligned}$$

Or, $32400 = 100\times4\times81$ and the required square root is equal to $10\times2\times9$, that is, 180.

Since a fraction is squared by squaring both numerator and denominator its square root is obtained by dividing the square root of the numerator by the square root of the denominator.

$$\sqrt{\frac{25}{121}} = \frac{\sqrt{25}}{\sqrt{121}} = \frac{5}{11}$$

B. Square Roots of Numbers not Easily Factored, Approximate Roots

$$\begin{aligned} 1^2 &= 1 \\ 10^2 &= 100 \\ 100^2 &= 10000 \\ 1000^2 &= 1000000 \\ 10000^2 &= 100000000 \end{aligned}$$

This shows that the square of a number less than 10, that is, a number of 1 digit, will consist of 1 digit or 2 digits, since 10 is the smallest number which will give 3 digits when squared.

The squares of numbers from 10 to 99 will give 3 digits or 4 digits. That is, the square of a number of 2 digits has either 3 digits or 4 digits. So also the square of a number of 3 digits will have 5 or 6 digits. Conversely, the square root of a number of 1 or 2 digits has 1 digit in its integral part. The square root of a number of 3 or 4 digits has 2 digits in its integral part. The square root of a number of 5 or 6 digits has 3 digits, etc. The square root of 5′76′48′01 has 4

digits The square of a unit's digit affects only the first two places. The square of a tens digit affects only the 3rd and 4th places, the square of a hundreds digit affects only the 5th and 6th places.

Example:

$$(3 \text{ tens})^2 = 30^2 = \underline{9}00$$
$$(7 \text{ tens})^2 = 70^2 = \underline{49}00$$
$$(6 \text{ hundreds})^2 = 600^2 = \underline{36}0000$$

Hence in taking the square root of a number by the formal process we divide the number off into periods of 2 digits each, *beginning at the decimal point.* We can then say how many digits there are in its square root and also say what the first digit must be.

Example: How many digits are there in the square root of 262144, and what is the first digit? Dividing off, 26′21′44, we see this comes from squaring a number of 3 digits. The 26 comes from squaring hundreds, so the first digit is 5. The number 26 is more than 5^2 but in forming the product we have hundreds multiplied by tens and units, which accounts for this.

The formal method of taking the square root is based on the algebraic form $(a+b)^2=a^2+2ab+b^2$. To extract the square root of $a^2+2ab+b^2$ we must discover a process that will yield $a+b$.

$$\begin{array}{l|l} \multicolumn{2}{l}{a^2+2ab+b^2 \,/\, \underline{a\quad}} \\ \multicolumn{2}{l}{\underline{a^2\qquad\qquad\qquad}} \\ 2a+b & +2ab+b^2 \end{array}$$

Evidently the square root must begin with a. Subtract the square of a and examine the remainder, to see how the second term is involved. It is clear that this second term may be obtained by division if we can get the correct divisor. The divisor that will give b as a quotient is $2a+b$, that is, the first

term doubled with the new term added. The new term will be unknown at this stage in working a problem so $2a$ is taken as a trial divisor. This goes $+b$ times. Put the $+b$ in the quotient and add it to the $2a$ and complete the division. $a+b$ is the square root.

Examples:

(1) To obtain the square root of 625 by this method:

	6′25 /25
	4
40	225
5	225
45	

Dividing off we see that the answer consists of 2 digits and the first digit is 2. This 2 means 20, since the square root has 2 digits. Subtract 20^2 omitting the zeros. Double the quotient (see $2a$. above), giving 40 as the trial divisor. 40 goes into 225, 5 times. Put 5 in the quotient and add it to the divisor. 5 times 45 gives 225.

(2) Find the square root of 531441.

	53′14′41 /729
	49
	414
142	284
1449	13041
	13041

There are 3 digits in the square root and the first digit is 7. Proceed as above to get the second digit, ignoring the last two digits in the meantime. This gives 72. Bring down the last period (41), and repeat the operation. Twice 720 is 1440, the trial divisor in the last division. This goes 9 times, giving 1449 as the complete divisor and 729 as the square root.

(3) Find the square root of 576.4801.

	5′76.48′01 /24.01
	4
44	176
	176
4801	4801
	4801

Obtain the 24 as before. Take in the next period (.48). What follows in the square root will evidently be a decimal. 480 into 48 goes 0 times. Put the zero in the quotient and bring down 01. 4800 into 4801 goes once, add the 1 and complete the division.

There is no finite number which when multiplied by itself will produce a number that is not a perfect square.

$(1.73)^2$ will give a result almost equal to 3. $(1.73205)^2$ will be much closer. $(1.7320508)^2$ is very nearly equal to 3 but not exactly. There is no finite number which will give exactly 3 when multiplied by itself. The quantities 1.73, 1.73205 and 1.7320508 are *approximate* square roots. Approximate roots may be found to any degree of accuracy specified. The process of finding approximate square roots for numbers not easily factored is the one given above.

Find the square root of .4, correct to 3 places of decimals.

$$\begin{array}{lrl}
 & .40'00'00..\text{etc.} & \\
 & \underline{36} & \\
123 & 400 & \underline{/.6324} = .632 \text{ correct to 3 places.} \\
 & \underline{369} & \\
1262 & 3100 & \\
 & \underline{2524} & \\
12644 & 57600 & \\
 & \underline{50576} & \\
 & 7024 &
\end{array}$$

Marking off and supplying zeros, use the method already described.

In taking the square root of a fraction, if the denominator be not a perfect square, either both numerator and denominator should be multiplied by a factor that will make the denominator a perfect square or the fraction should be reduced to a decimal fraction before taking its square root.

$$\sqrt{\tfrac{5}{8}}=\sqrt{\tfrac{10}{16}}=\frac{\sqrt{10}}{4}$$

To complete, it will be necessary to find the square root of 10 (approximately) and divide it by 4.

Or, $\sqrt{\tfrac{5}{8}}=\sqrt{.625}$, which may be completed in one operation.

C. Roots of a Higher Order

Cube roots of perfect cubes may be obtained by the method of prime factors or cubic factors. It is desirable that the cubes of numbers from 1 to 12 be memorized. These in order are: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728.

The formal method of extracting a cube root is difficult and tedious. As cube roots can be readily found by logarithms the other method is unnecessary.

To obtain the cube root of 46656 by factoring:

$$46656 = 2\times2\times2\times2\times2\times2\times3\times3\times3\times3\times3\times3$$
$$= (2\times2\times3\times3)^3 = 36^3. \quad \text{Cube root is } 36.$$

Or,

$$\begin{array}{r|l} 8 & 46656 \\ \hline 8 & 5832 \\ \hline 9 & 729 \\ \hline 3 & 81 \\ \hline & 27 \end{array}$$

This gives $(8\times8)\times(9\times3)\times27$.

Cube root is $4\times3\times3=36$.

Other roots may be obtained by factoring. It is assumed here that a cube root is one of the 3 equal factors producing a number, a fourth root one of its 4 equal factors, etc.

Also, a fourth root is the square root of a square root and a sixth root is the cube root of a square root.

Exercise VIIa

(1) Find the square roots of the following numbers: 289, 441, 1369, 3481, 5329, 6084, 7396, 8281, 15129, 54756, 92416, 370881, 801025, 516961, 426409.

(2) Find by factoring the square roots of:
1764, 1089, 3969, 6084, 50625, $21\times8\times6\times7$.

(3) Find the value of

$$\sqrt{1\tfrac{9}{16}},\ \sqrt{1\tfrac{25}{144}},\ \sqrt{1\tfrac{64}{225}},\ \sqrt{4\tfrac{25}{36}},\ \sqrt{9\tfrac{49}{64}}.$$

(4) State orally the square roots of the following numbers:
14400, 160000, 2250000, .49, 1.44, .0081, .0004, .0144, .000121, .000001.

(5) Find to two places of decimals the square roots of: 2, 5, 20, 200, 3, 30, 48, 13, 130, 182, 419, 5472, 6479, 7963, 9984.

(6) Extract the square roots of:

32.49, 1.4161, 12.8164, 3782.25, 100.2001, .4225, .001369, .017161, .00822649, .000729, 103.4289, .597529.

(7) By first reducing to a decimal, find the square roots of the following fractions correct to two decimal places:

$$\tfrac{1}{3},\ 1\tfrac{1}{4},\ \tfrac{3}{8},\ \tfrac{10}{17},\ 4\tfrac{2}{5},\ 15\tfrac{1}{4},\ 7\tfrac{8}{9}.$$

(8) By first making the denominator a perfect square, find the square roots of the following fractions correct to two decimal places:

$$\tfrac{27}{32},\ \tfrac{5}{12},\ \tfrac{9}{13},\ 3\tfrac{1}{3},\ 4\tfrac{2}{5},\ 1\tfrac{1}{2},\ 6\tfrac{2}{3},\ \tfrac{5}{11},\ \tfrac{17}{24},\ \tfrac{15}{28}.$$

(9) Find the square roots of the following numbers to the degree of accuracy indicated:

8057430 correct to 2 places of decimals
27784.7 correct to 2 places of decimals
277847 correct to 2 places of decimals
307.767 correct to 2 places of decimals
3077.67 correct to 3 places of decimals
3 correct to 7 places of decimals
2 correct to 5 places of decimals
5 correct to 3 places of decimals

Exercise VIIb

(1) Test the following relations by working to three places of decimals:

$$\sqrt{2}\times\sqrt{3}=\sqrt{6},\ \ \sqrt{5}\times\sqrt{7}=\sqrt{35},\ \ \sqrt{12}\div\sqrt{4}=\sqrt{3},$$
$$\sqrt{48}=4\sqrt{3},\ \ \sqrt{50}=5\sqrt{2}.$$

(2) Show that $3+\sqrt{2}$ is the square root of $11+6\sqrt{2}$.

(3) Find the difference between $(\sqrt{2}+\sqrt{5})^2$ and $2+5$.

(4) Find the difference between $(\sqrt{6}-\sqrt{3})^2$ and $6-3$.

(5) Find the fourth roots of:

20736, 6561, 194481.

(6) Find, to the nearest tenth, the fourth root of 5.

(7) Calculate in yards, to two decimal places, the side of a square field whose area is: 2 acres; 100 acres; $\frac{1}{2}$ sq. mile.

(8) How many yards of fencing are required to enclose a square field of 10 acres?

(9) A square organ pipe 16 ft. long has 12 cu. ft. of air space. Find, to the nearest tenth of an inch, the dimensions of the cross-section of this pipe.

(10) A hallway having an area of 108 sq. ft. is three times as long as it is wide. Find the dimensions.

(11) How many rods of fence are required to enclose a rectangular field of 20 acres whose length is twice its width?

(12) The area of a rectangle is 700 sq. in., and its sides are in the ratio of 4 : 7. What are the lengths of the sides?

(13) The length of a room is $1\frac{1}{4}$ times its breadth, and the breadth is $1\frac{1}{3}$ times its height. The room contains 1620 cu. ft. Find its dimensions.

CHAPTER VIII

MENSURATION—LINES AND AREAS

A. Lines

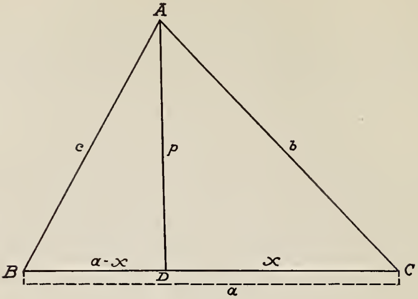
The square on the hypotenuse of a right angled triangle is equal to the sum of the squares on the other two sides. This theorem is established in Geometry (Theorem of Pythagoras). If the base of a right angled triangle be b units, the perpendicular p units and the hypotenuse h units, then:

$$h^2=b^2+p^2, \text{ or } b^2=h^2-p^2, \; p^2=h^2-b^2.$$

The ratio of the circumference of a circle to its diameter is more difficult to establish exactly but it has been calculated to many places of decimals. If the circumference be c units and the diameter d units, $\frac{c}{d}=3.14159265\ldots$

The value 3.14159 is accurate enough for the majority of problems, 3.1416 is fairly close and $3\frac{1}{7}$ is a useful approximation. This ratio may be roughly verified by measuring the circumferences and diameters of cylinders by means of a string. For the purpose of reducing this relation to a simple formula, and for theoretical discussions the ratio $c : d$ is expressed by the Greek letter π. $c=\pi d$ or $c=2\pi r$, the radius being r units long.

To find the lengths of the segments into which the perpendicular from the vertex divides the base of a triangle, also the length of this perpendicular.



In triangle ABC, denote BC by a, AC by b, AB by c, AD by p and DC by x.

Then BD is $a-x$.

$p^2=b^2-x^2$ in triangle ADC.

$p^2=c^2-(a-x)^2$ in triangle ADB.

Then $b^2-x^2=c^2-(a-x)^2$

$$=c^2-a^2-x^2+2ax.$$

Transposing, $2ax=a^2+b^2-c^2$

$$x=\frac{a^2+b^2-c^2}{2a}=\text{DC}.$$

$$\text{DB}=a-\left\{\frac{a^2+b^2-c^2}{2a}\right\}$$

$$=\frac{a^2+c^2-b^2}{2a}$$

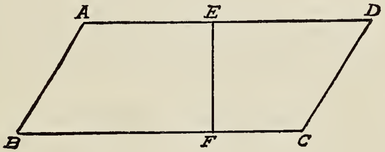
$$p^2=b^2-x^2=b^2-\left\{\frac{a^2+b^2-c^2}{2a}\right\}^2$$

For particular values of a, b and c these lengths may be easily found.

B. Areas of Surfaces

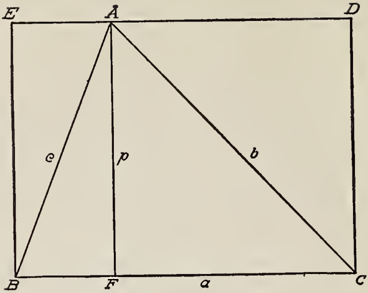
Rectangle. The number of units in the length of a rectangle multiplied by the number of units in its width is equal to the number of square units in its area. This is generally abbreviated to the form: area = length $\times$ breadth, or area = base $\times$ altitude.

Parallelogram. In the parallelogram ABCD, EF is drawn perpendicular to the parallel sides. If the figure ABFE be removed and placed so that AB coincides with



DC, the figure becomes a rectangle with area = BC $\times$ EF. Then the area of the parallelogram = BC $\times$ EF = base $\times$ altitude.

Triangle. If ED be drawn through the vertex A of the triangle ABC to meet perpendiculars from B and C at E and D, EBCD is a rectangle. Since the triangle EBA = triangle FBA, and triangle DCA = triangle FCA, the triangle ABC is $\frac{1}{2}$ the rectangle EBCD. But the area of EBCD = BC $\times$ AF. Then the area of triangle ABC = $\frac{1}{2}$BC $\times$ AF = $\frac{1}{2}$ base $\times$ altitude.



If AB $=c$ units, AC $=b$ units, BC $=a$ units and AF $=p$ units, it has been shown that $p^2=b^2-\left\{\frac{a^2+b^2-c^2}{2a}\right\}^2$

then the area $=\frac{1}{2}$ BC $\times p=\frac{1}{2}ap=\frac{1}{2}a\sqrt{b^2-\left\{\frac{a^2+b^2-c^2}{2a}\right\}^2}$

which may be reduced to the form

$$\sqrt{s(s-a)\ (s-b)\ (s-c)}$$

where $2s=a+b+c$.

$$\text{Area}=\tfrac{1}{2}a\sqrt{\frac{4a^2b^2-(a^2+b^2-c^2)^2}{4a^2}}$$

$$=\sqrt{\tfrac{1}{4}a^2\left\{\frac{4a^2b^2-(a^2+b^2-c^2)^2}{4a^2}\right\}}$$

$$=\sqrt{\frac{(2ab+a^2+b^2-c^2)\quad(2ab-a^2-b^2+c^2)}{16}}$$

$$=\sqrt{\frac{(a+b+c)}{2}\ \frac{(a+b-c)}{2}\ \frac{(c+a-b)}{2}\ \frac{(c-a+b)}{2}}$$

$$\left.\begin{aligned} \text{If } a+b+c&=2s \\ a+b-c&=2s-2c \\ a-b+c&=2s-2b \\ b+c-a&=2s-2a \end{aligned}\right\} \begin{aligned} \text{Area} &= \sqrt{\frac{2s}{2}.\frac{2s-2c}{2}.\frac{2s-2b}{2}.\frac{2s-2a}{2}.} \\ &= \sqrt{s(s-a)\ (s-b)\ (s-c)} \end{aligned}$$

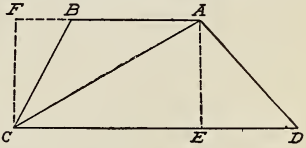
If $b=c=a$ the triangle is equilateral, $s=3\frac{a}{2}$ and the area

$$=\sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}}=\sqrt{\frac{3a^4}{16}}=\frac{a^2}{4}\sqrt{3}=a^2\times\frac{1.73205}{4}=a^2(.433)$$

To find the area of an equilateral triangle multiply the square of the side by .433.

Trapezium. The trapezium has one pair only of opposite sides parallel.

In ABCD let AB and CD be parallel. Divide the figure into two triangles by joining AC.



Area of trapezium = area of triangle ADC + area of triangle ABC.

Area of triangle ABC $=\frac{1}{2}$AB.CF $=\frac{1}{2}$AB.AE.

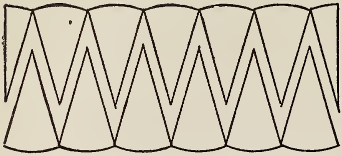
Area of triangle ADC $=\frac{1}{2}$CD.AE.

Total area $=\frac{1}{2}$AB.AE $+\frac{1}{2}$CD.AE.

$=\frac{1}{2}$(AB+CD)AE = half the sum of the parallel sides $\times$ distance between them.

Circle. If a circle be divided into two semicircles and each semicircle be divided into a large number of sectors, as shown in the figure, then straightening out

each semicircle by bending at the ends of the small



arcs, the sectors of one semicircle may be fitted into those of the other to form an approximate rectangle. If the number of sectors be made infinitely large the arcs will form a straight line and the figure will differ from a rectangle by a quantity too small to measure. Area of rectangle = base × altitude. But the base is half the circumference and the altitude is the radius.

Area of circle $= \frac{1}{2}c \times r = \frac{1}{8}(2\pi r)r = \pi r^2.$

Sector of a Circle. A sector of a circle may be divided into two equal sectors, and each of these halves divided as in the case of the circle and the parts fitted together in the same way. The result is a rectangle with base equal to half the arc and altitude equal to the radius.

Then area of sector $= \frac{1}{2}$ arc × radius.

If the angle of the sector is given in place of the arc or radius, the area of the sector may be found as a fractional part of the whole circle. The area of the sector is the same fraction of the area of the circle as the angle of the sector is of 360°.

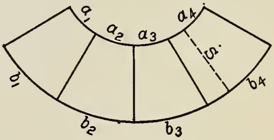
Cylinder. If the opposite edges of a rectangular piece of paper be brought together it may be made to enclose a space in the form of a cylinder. The area of the paper = length × breadth. The length becomes the circumference of the end of the cylinder and the breadth becomes the length of the cylinder (or vice versa). Then the area of the curved surface of a cylinder = the circumference of the end × the length. The

ends of the cylinder are circles and their areas may be found if required.

Cone. If a piece of paper be cut in the form of a sector of a circle and the two edges representing radii be brought together the figure enclosed may be made conical. The area of the sector $= \frac{1}{2}$ arc $\times$ radius. The arc becomes the circumference of the base and the radius the slant height. Then the area of the curved surface of a cone $= \frac{1}{2}$ circumference of base $\times$ slant height, or, circumference of base $\times$ half the slant height.

Pyramid. The sides of a pyramid are triangles and the area of each $= \frac{1}{2}$ base $\times$ altitude. Care must be exercised to see that the altitude taken is the altitude of the triangle, that is, the length of the perpendicular dropped from the vertex to the base of the triangle. If the base of the pyramid be one of the plane geometrical figures we have considered, or if it may be divided into such figures, its area may be found if necessary.

Frustum of pyramid or cone. The slant sides of the frustum of a pyramid are trapeziums and their areas may be found by the rule already worked out. In the case of the frustum of a cone the band of curved surface surrounding it may be divided into a very great number of parts, each part approximately a trapezium. Let the arcs of the smaller circle at one edge of the band be



a_1, a_2, a_3, etc., and the corresponding arcs of the larger circle be b_1, b_2, b_3, etc. If the slant height of the frustum be s units, then the distance between the parallel sides of each trapezium will be S.

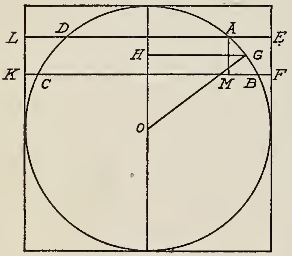
Total area

$$=\tfrac{1}{2}(a_1+b_1)s+\tfrac{1}{2}(a_2+b_2)s+\tfrac{1}{2}(a_3+b_3)s+\text{etc.}$$
$$=\tfrac{1}{2}s(a_1+b_1+a_2+b_2+a_3+b_3+\text{etc.})$$
$$=\tfrac{1}{2}s(a_1+a_2+a_3+\ \ldots\ +b_1+b_2+b_3\ \ldots)$$
$$=\tfrac{1}{2}s(\text{shorter circumference}+\text{ longer circumference})$$

or the area = the sum of the two circumferences × half the slant height = $\frac{1}{2}s$ (C+c).

Evidently the area of a circular ring or annulus, such as a path of constant width surrounding a circular area may be found in the same way.

Sphere. Assuming that the sides of similar triangles are proportionals, the area of the surface of a sphere may be proved equal to the area of the curved surface of a cylinder of the same diameter and with height equal to the diameter.



The sphere will evidently fit the cylinder as shown in the figure. Suppose both sphere and cylinder be cut into slices by planes parallel to the ends of the cylinder. EFKL is a section of such a slice of the cylinder and ABCD of the corresponding slice of the sphere.

If the slices be made very thin AB will be approximately a straight line. The area of the circular band surrounding the sphere will be $\frac{1}{2}$ the sum of the two circumferences × slant height = average of the two circumferences × slant height.

Suppose the average circumference passes through G, then the area = circumference through G×AB. =

2π HG.AB. But the triangles OHG and BMA are similar, since they are equiangular. Then HG/OG = AM/AB and HG.AB = AM.OG.

Area of the band = 2π HG.AB = 2π AM.OG. But 2π OG is the circumference of the sphere, or cylinder. Hence the area of the band on the surface of the sphere = circumference of cylinder $\times$ AM = circumference $\times$ EF = the area of the corresponding band on the cylinder.

Adding the bands composing the total surface area of the sphere we get the sum of the bands on the cylinder, that, is, its curved surface.

Area = $2\pi r \times$ height = $2\pi r \times 2r = 4\pi r^2$.

Similar Figures

The properties of similar figures, as established in Geometry, may be used to advantage in mensuration problems. Some of these properties are:

The diagonals of two squares have the same ratio as the sides of the squares.

The linear dimensions of a polygon bear a constant ratio to corresponding linear dimensions of a similar polygon. Rectangles, triangles, parallelograms, etc., are polygons in this connection.

The circumferences of two circles have the same ratio as their radii or diameters.

The areas of two squares have the same ratio as the squares on their diagonals.

The areas of two polygons have the same ratio as the squares on corresponding linear dimensions.

The areas of two circles have the same ratio as the squares of the lengths of their radii, of their diameters, or of their circumferences.

These may be verified by calculation in particular cases.

Summary

(1) In a right angled triangle, $h^2 = p^2 + b^2$.

(2) In a circle, $c = \pi d = 2\pi r$.

(3) Area of a rectangle = base × altitude = length × breadth.

(4) Area of a parallelogram = base × altitude.

(5) Area of a triangle = base × half the altitude = $\sqrt{s(s-a)(s-b)(s-c)}$.

(6) Area of a trapezium = half the sum of the parallel sides × the perpendicular distance between them.

(7) Area of a circle $= \pi r^2$.

(8) Area of a sector of a circle = half the arc × the radius.

(9) Area of the curved surface of a cylinder = circumference × length.

(10) Area of curved surface of a cone = circumference of the base × half the slant height.

(11) Area of curved surface of frustum of a cone, or of a circular ring = half the sum of the two circumferences × distance between them.

(12) Area of the surface of a sphere $= 4\pi r^2$.

Exercise VIIIa

(Use the approximation $3\frac{1}{7}$ for π)

(1) The sides of a right angled triangle are 3.9 in. and 5.2 in., respectively. Find the length of the hypotenuse.

(2) In a right angled triangle the hypotenuse and one side measure respectively, 27 m., and 19 m. Find the length of the remaining side.

(3) The gable of a house is 24 ft. wide and 8 ft. high from the plate to the ridgepole. How long must the rafters be cut if they are to project one foot over the eaves? Verify your result by drawing to scale.

(4) Find the altitude of an equilateral triangle each of whose sides is "a" inches.

(5) The hypotenuse of a right angled triangle with equal sides is 16 inches. Find the lengths of the two equal sides.

(6) Find, in yards, the diameter of a circular running track measuring 8 laps to the mile.

(7) A wheel of diameter 6 ft. making 50 revolutions per minute, is connected by a belt to a wheel of 1 ft. 6 in. diameter. How many revolutions will the latter wheel make in a minute?

(8) A derrick is 48 ft. high, and is supported by three steel cables each reaching from the top of the derrick to a stake in the ground 45 ft. from the foot of the derrick. How much steel cable does it take, allowing 10 ft. for fastening all three cables?

(9) The side of a rhombus is 13 inches long and one diagonal is 24 inches long. Find the length of the other diagonal.

(10) One ship sails 5 miles due west and then 4 miles due north. Another ship starts at the same place and sails 6 miles due north and then $3\frac{1}{2}$ miles due west. How far are the ships now apart?

(11) How many revolutions will a bicycle wheel of 28 inches in diameter make in travelling one mile?

(12) A flower bed is in the form of a rectangle with semicircles added on at the ends. If the straight sides are 15 ft. long and the width 6 ft., find the length of the perimeter of the bed.

(13) The driving wheel of a locomotive engine is 5 ft. in diameter. It turned 2500 times in going 7 miles. Find, in yards, what distance is lost owing to the slipping of the wheel on the rail.

(14) An engine transmits power to a mill by means of a belt. The driving wheel is 4 ft. in radius and makes 60 complete revolutions in 2 min. 12 sec. The driven wheel has a radius of 3 ft.

(1) Find the angle turned through by the driving wheel in 4 sec.

(2) Find the number of revolutions made by the driven wheel in 1 minute, assuming that there is no slipping of the belt.

(15) The perimeter of a semicircle is 48 inches. Find its radius.

(16) Find, in inches, the side of the greatest square piece of timber that can be cut from a tree whose circumference is 8 ft.

(17) The sides of a triangle are 13 in., 14 in., and 15 in., respectively. Find the length of the perpendicular on the 14 in. side from the opposite angle.

(18) Find the length of an arc which subtends an angle of $72°$ at the centre of a circle of radius $4\frac{2}{3}$ inches.

(19) What is the length of the side of the greatest cube which can be cut from a sphere 1 m. in diameter?

(20) Two equal circles with radii 10.5 in., pass each through the centre of the other. Find the perimeter of the area common to the two circles.

(21) The sides of a right angled triangle are respectively 7 ft. and 24 ft. Find the length of the perpendicular on the hypotenuse from the opposite vertex.

(22) The radius of a circle is 126 inches. Find the length of a tangent to the circle drawn from a point 130 inches from the centre.

(23) Find the side of the largest square that can be inscribed in a circle $12\frac{5}{6}$ ft. in circumference.

(24) A smoke stack built on level ground has guy wires anchored at points 80 ft. distant from its base. The wires are 120 ft. long. At what height above the base are the wires attached to it?

(25) A crane, with a horizontal tie-rod 14 ft. long, raises a packing case from a vessel alongside a dock through a vertical distance of 25 ft., then swings it through an angle of 90° and finally lowers it 5 ft. to the dock. Find the total distance through which the packing case has been moved.

Exercise VIIIb

(1) What is the cost of a triangular piece of land with a base of 2112 ft. and an altitude of 1650 ft. at $45 per acre?

(2) Find the number of acres in a triangular field whose sides are 325 ft., 350 ft., and 375 ft.?

(3) Find the areas of the circles whose radii measure: 10.5 m., 3.5 in., 2.1 ft.

(4) Show that the area of a semicircle described on the hypotenuse of a right angled triangle is equal to the sum of the semicircles described on the two sides of the triangle.

(5) Find the area of a circle whose circumference measures $5\frac{1}{2}$ ft.

(6) The perimeter of a semicircle is 108 inches. Find its area.

(7) An equilateral triangle and a square have the same perimeter. Find the ratio of their areas.

(8) How much more will it cost at $1.25 per rod to build a fence around 80 acres in the form of a square than in the form of a circle?

(9) What is the difference in area between a circle of radius 27.3 cm. and a square with side equal to the diameter of the circle?

(10) An equilateral triangle and a circle have each a perimeter of 48 in. Find the difference in their areas.

(11) A circular race course contains $61{,}615\frac{5}{7}$ sq. yds. If the inner diameter is 320 yd., find the width of the course.

(12) The area of a circle is 3850 sq. ft. and the area of a sector of the same circle is 1540 sq. ft. Find the length of the arc of the sector.

(13) A road runs around a circular pond. The outer circumference is 440 yd., and width of the road is 20 yd. Find the area of the pond.

(14) The common chord of two equal circles of radius 1.54 m. is equal to the radius. Find the area common to the two circles.

(15) A regular hexagon is inscribed in a circle of radius 15.4 cm. What is the area of that part of the circle without the hexagon?

(16) A semicircle and an isosceles triangle have equal bases and equal altitudes. Show that their areas are as $\pi : 2$.

(17) The perimeter of an equilateral triangle is 60 cm. Find its area.

(18) Find, in acres, the area of a path 1.5 metres wide around a circular pond whose diameter is 50 metres.

(19) A running track is 440 yd. long (measured along the inner edge). It consists of two straight pieces and two semicircles, each 110 yd. long. The track is 15 ft. wide. Find the cost of covering it with cinders at 15c per sq. yd.

(20) The total area of three circles is 60.445 sq. m., and their radii are proportional to 4, 5 and 6. Find the area of each circle.

(21) The electrical resistance of a copper wire varies inversely as the area of its cross section. A wire with a diameter of 2 mm. has a resistance of 6.57 ohms. Find the resistance of a wire of diameter 3 mm.

Exercise VIIIc

(1) A tin plate measuring 18 in. by $12\frac{1}{2}$ in., costs 15c. What is the cost per square yard?

(2) Find the number of sq. cm. in a sq. in., given 1 in. = 2.54 cm. (correct to three places).

(3) A book contains 360 pages which measure 8 in. by $5\frac{1}{2}$ in. Every page has a blank margin 1 in. deep along each short side and $\frac{3}{4}$ in. deep along each long side. Find the total area available for printing.

(4) A street half a mile long and 60 feet wide is paved with granite blocks. If 30 blocks are required to pave one square yard, how many will be required to pave the whole street?

(5) A rectangular grass plot, 32 yd. 1 ft. long by 20 yd. 2 ft. 6 in. broad, is surrounded by a path 10 ft. wide along the two longer sides, and 6 ft. 3 in. wide along the two shorter sides. Find the area of the path.

(6) On the floor of a square room a margin 4 ft. 6 in. wide is left uncarpeted. The cost of polishing this at 50c per sq. yd. is $10.50. Find the size of the room.

(7) The length of a field is to its width as 3 :2. Its area is 15 acres. Find the length of the field.

(8) A four-foot sidewalk of 2 inch planks is laid around a square athletic field containing 10 acres. The inner edge of the sidewalk is two feet out from the boundary of the field. What did the planking cost at $40 per M?

(9) A city bought a piece of land for a public playground. The parallel sides of this plot are 460 ft. and 620 ft., and its width is 308 ft. What did this piece of land cost at $125 per acre?

(10) How many feet of lumber are there in a two inch plank 18 ft. long, 16 in. wide at one end and 12 inches at the other?

(11) How many acres are there in a piece of land in the form of a parallelogram 68 rd. long and 54 rd. wide?

(12) The area of a trapezium is 300 sq. in. The perpendicular distance between the two parallel sides is 12 in. If the length of one of the parallel sides is 30 in., what is the length of the other?

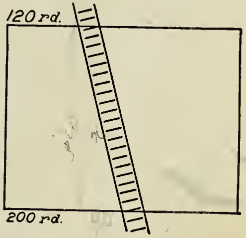
(13) A farmer had a rectangular field which was twice as long as it was wide and contained 20 acres. He divided it into two square fields and fenced them at 50c a yard. How much did the fence cost him?

(14) A square field contains 48,400 square metres. Find the length of the side in kilometres.

(15) The parallel sides AB and CD of a trapezium ABDC are respectively 10 ft. and 18 ft. long, and the non parallel sides CA and DB are respectively 15 ft. and 12 ft. long. CA and DB are produced to meet at O. Find the lengths of OA and OB.

(16) The length of a rectangular field is to its breadth as 5 : 4. One-fifth of the area is wooded, and the remainder, 576 sq. rd. is under cultivation. What are the dimensions of the field?

(17) A railroad cuts through a section of land as shown in the diagram and uses a strip 4 rd. wide. Find the land damage for the right of way at $40 an acre.



(18) In measuring the side of a square field, the side is taken 2 chains too long, and the calculated area is

consequently 36 acres, 4 sq. chains too large. What is the true area of the field?

(19) Find the area of a trapezium whose parallel sides are 84 ft. and 177 ft. in length and whose non parallel sides are 34 ft. and 65 ft. in length.

(20) Find the area of an isosceles trapezium, if the parallel sides are 10 ft. and 18 ft. long and the non parallel sides each 8 ft. long.

(21) The diagonal of a four-sided field is 96 rd. and the perpendiculars on it from the opposite corners are 45 rd. and 64 rd. respectively. Find the number of acres in the field.

(22) A park is 80 rd. long and 56 rd. wide. Around the outside of it is a brick walk 8 ft. wide. The exposed surface of each brick is 4 in. by 8 in. Find the cost of the brick at $8.50 per thousand.

(23) A body of men is drawn up in a hollow square four deep with 85 men in a side of the square. Show that they can also be drawn up in a solid square, and find the number of men there will then be in a side.

(24) If a wheat drill is 8 ft. wide and is adjusted to sow $1\frac{1}{2}$ bushels to the acre, how much seed will be used in crossing a field 22 times, the width of the field being 120 rd.?

(25) A tennis court is in the middle of a grass plot. The dimensions of the tennis court are 78 ft. by 36 ft., and all round the court is left a border of grass which is 12 ft. wide. What would be the cost of erecting wire netting around this plot if the netting was 12 ft. high and cost $4.50 per roll of 50 yards, the width of the roll being 6 ft.?

Exercise VIIId

(1) A cylindrical tank is 30 inches long and 10.5 inches in diameter. Find the area of its entire surface.

(2) A sheet of paper 24.5 cm. wide and 30.24 cm. long is rolled to form a cylinder with the two longer edges just meeting. Find the diameter of the end of this cylinder. Find the area of the curved surface.

(3) A roller, 1 ft. 2 in. in diameter and 3 ft. long, makes 36 revolutions in passing from end to end of a lawn. Find, in square yards, the area rolled when it has passed from end to end 48 times.

(4) What is the area of the entire cooling surface of an ice cream freezer 6 inches in diameter and 18 inches long?

(5) The height of a triangular prism is 12 inches and the base is a right angled triangle whose sides are 6 inches and 8 inches. Find the area of its entire surface.

(6) A V-shaped trough is 16 ft. long, 12 in. wide at the top and 8 in. deep. Find the cost of painting the entire surface at 15c per square yard.

(7) The area of the curved surface of a cylinder is 135 sq. cm. and its height is 4.5 cm.; find the radius of its base.

(8) The area of the curved surface of a cylinder is to the total area of its surface as 3 : 4. Show that the radius of the end is to the length of the cylinder as 1 : 3.

(9) Find the area of the surface of a sphere whose diameter is 24.5 cm.

(10) A hemisphere, a cylinder, and a cone stand on equal bases. If their heights are the same, compare the areas of their curved surfaces.

(11) Find the area of the canvas required for 50 conical tents, the diameter of the base of each being 14 ft. and the height 12 ft.

(12) A church spire is in the form of an octagonal pyramid having a base 6 feet on a side and a slant height of 40 feet. Find the cost of roofing at 75 cents per square yard.

(13) Find the lateral surface of the frustum of a square pyramid whose base is 3 ft. square and top $2\frac{1}{2}$ ft. square, the slant height being 2 ft.

(14) A cube and a sphere have equal surfaces. Find the ratio of the edge of the cube to the diameter of the sphere.

(15) Find the lateral surface of the frustum of a cone whose ends are 6 ft. and 4 ft. in diameter and whose slant height is 12 ft.

(16) The height of a cylinder is 14 inches and its curved surface 132 square inches. Find its total surface.

(17) A frustum of a cone has the following dimensions: diameter of base 14 in., diameter of top 8 in., perpendicular height 5 in. Find the total area of its surface.

(18) One sphere has twice the diameter of another sphere. What is the ratio of their surfaces?

(19) How many square inches of tin will be required to make a vessel (without a cover) with a base 2 ft. in diameter, a top 6 in. in diameter, and a depth of 15 in.?

(20) A 14-inch globe represents the earth, which may be considered a sphere, with an approximate radius of 3920 miles. Find the scale in miles per inch by which its surface is represented on the globe.

(21) The area of the lateral surface of a cone is 484 sq. in., and its slant height is 56 in. Find the radius of its base.

CHAPTER IX

MENSURATION—VOLUMES

A. Solids of Constant Cross Section with End Surfaces Parallel

Rectangular Solid. If the area of the base of a rectangular solid be A square units, then the volume of the solid on this base and of one unit thickness (or height), will be A cubic units, since each square unit of surface of the base is the face of a cubic unit (1 unit each way). If the height of the solid be c units, then the volume must be Ac cubic units. But if the base be a units by b units, the area of the base is ab square units and the volume of the solid is abc cubic units. Volume of a rectangular solid = area of base × height, or, length × breadth × thickness.

Prism. If a rectangular solid be split vertically by a plane which contains the diagonal of the base, the result will be two equal triangular prisms, each on half the base of the rectangular solid and each half the volume of the solid. The new base × height will give half the original volume and therefore the volume of the prism. Volume of a prism = area of base (end) × perpendicular height (length).

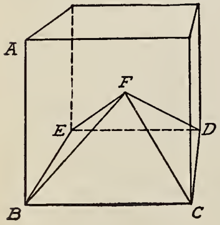
The reasoning applied to the rectangular solid above however, applies with equal force to the prism. If the base of the prism contains A square units, a portion of the prism one unit high will contain a cubic unit on each square unit of base, or A cubic units. A prism h units high will contain Ah cubic units, the formula given above. This applies equally to prisms with bases other than triangular.

Cylinder. If the base of the prism be a regular polygon, and the number of its sides be increased indefinitely, the prism will approach the form of a cylinder and its volume will differ from the volume of the cylinder by a quantity too small to be measured. Hence the volume of a cylinder = area of base (end) × perpendicular height (length).

The reasoning applied to the rectangular solid and prism gives the same result independently. The same reasoning will apply to any solid having a constant cross section and with plane parallel end surfaces. Although we are considering here only those solids whose end surfaces are perpendicular to the side surfaces the formula will apply also to solids whose side surfaces may not be perpendicular to the end surfaces.

The volume of the material used in making a box, with or without cover, is often more easily obtained by subtracting from the total volume enclosed by the outside surfaces, the volume of the cavity inside, than by attempting to obtain the volumes of the different side pieces, allowing for corners, etc., and adding.

B. Solids with Varying Cross Section



Pyramid. Let AD be a cube, with F the point of intersection of the lines joining opposite angular points. F is the central point of the cube. F is also the vertex of a square pyramid on the base BCDE. Another such pyramid stands on each of the other 5 faces of the cube. The cube then contains 6 of such pyramids.

The area of the base of the cube $\times$ height of the cube $=$ volume of cube $=$ 6 times volume of pyramid.

The area of the same base $\times$ height of pyramid $= \frac{1}{2}$ volume of cube (since height of pyramid $= \frac{1}{2}$ height of cube) $=$ 3 times volume of pyramid. That is, the base of the pyramid (same as cube) $\times$ height of pyramid $=$ 3 times volume of pyramid. Then the volume of a pyramid $=$ area of base $\times \frac{1}{3}$ height.

Just as the rectangular solid was split into prisms the square pyramid may be split into triangular pyramids etc., the volume of each part being the area of its base $\times \frac{1}{3}$ height.

Cone. Since the volume of any pyramid $=$ area of base $\times \frac{1}{3}$ perpendicular height, we may take a pyramid with base any regular polygon. By increasing indefinitely the number of the sides of the polygon the volume of the pyramid finally differs from the volume of a cone by less than any assignable quantity, hence, volume of a cone $=$ area of base $\times \frac{1}{3}$ perpendicular height.

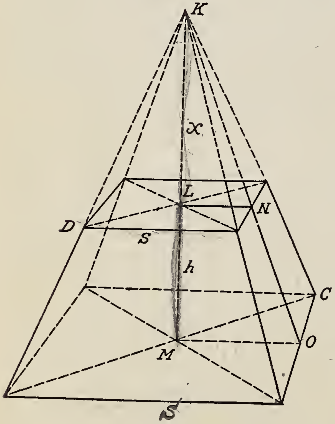
Sphere. If the surface of a sphere be divided into an infinitely large number of small areas, each area may be regarded as a plane surface and the base of a pyramid with its vertex at the centre of the sphere. Suppose s_1, s_2, s_3, s_4, etc., be the areas of these surfaces, and r the radius of the sphere, the total volume of the pyramids would be $\frac{1}{3}(s_1r+s_2r+s_3r+s_4r+\text{etc.}) = \frac{1}{3}r$ $(s_1+s_2+s_3+s_4+\text{etc.})$. $s_1+s_2+s_3+s_4+\text{etc.}$ . . . would be the area of the surface of the sphere and the sum of the volumes would be the volume of the sphere, then,

$$\text{Volume of sphere} = \tfrac{1}{3}r\ (\text{area of surface})$$
$$= \tfrac{1}{3}r(4\pi r^2) = 4\ \pi r^3.$$

C. Incomplete Solid Figures

Frustum of Pyramid. If a pyramid be cut by a plane parallel to its base the part remaining after the removal of its top portion is called a frustum. Its volume will be the volume of the whole pyramid diminished by the volume of the part removed. In most cases only the height of the frustum is given. The chief difficulty in finding the volume lies in the calculation necessary to find the height of the portion cut off.

DC is a frustum of a pyramid the vertex of which would be at K. Suppose the length of the side of the base of the frustum be S units and of the top s units. Let x units be the height of the top portion. Then x+h units would be the height of the complete pyramid, if h is the height of the frustum.



Comparing similar triangles in the figure, KLN and KMO.

$$\frac{x}{x+h}=\frac{\frac{1}{2}s}{\frac{1}{2}S}=\frac{s}{S}$$

$Sx=sx+sh.\quad Sx-sx=sh.$

$x=\dfrac{sh}{S-s}=$ height of part cut off.

$$x+h=\frac{sh}{S-s}+h=\frac{Sh}{S-s}$$

$=$ height of complete pyramid.

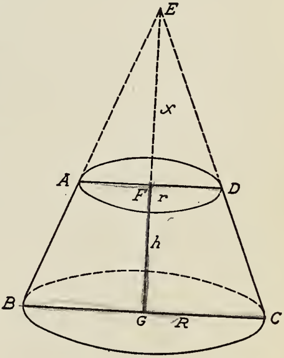
Volume of frustum $=\dfrac{1}{3}\,S^2(x+h)-\dfrac{1}{3}\,s^2(x)$

$$=\frac{1}{3}S^2\left(\frac{Sh}{S-s}\right)-\frac{1}{3}s^2\left(\frac{sh}{S-s}\right)$$

$$=\frac{1}{3}h\left(\frac{S^3-s^3}{S-s}\right)=\frac{1}{3}h(S^2+Ss+s^2).$$

Frustum of Cone. The method employed in finding the volume of a frustum of a cone is the same as that used in finding the volume of a frustum of a pyramid.

Let $ABCD$ be a section through the centre of the part of a cone left after removing the top.



Let R be the longer radius and r the shorter radius, h the height of the frustum and E the vertex of the complete cone.

Suppose the top is x units high. Comparing triangles EFD and EGC,

$$\frac{x}{x+h}=\frac{r}{R} \qquad xR=xr+rh$$

$$x(R-r)=rh$$

$$x=\frac{rh}{R-r}=\text{height of part removed.}$$

$$x+h=\frac{rh}{R-r}+h=\frac{Rh}{R-r} \quad =\text{height of complete}$$

cone.

Volume of frustum $=\frac{1}{3}\pi R^2(x+h)-\frac{1}{3}\pi r^2(x)$

$$=\frac{1}{3}\pi R^2\left(\frac{Rh}{R-r}\right)-\frac{1}{3}\pi r^2\left(\frac{rh}{R-r}\right)$$

$$=\frac{1}{3}\pi\frac{R^3h}{R-r}-\frac{1}{3}\pi\frac{r^3h}{R-r}=\frac{1}{3}\pi h\left(\frac{R^3-r^3}{R-r}\right)$$

$$=\frac{1}{3}\pi h(R^2+Rr+r^2).$$

Similar Solids. The volumes of similar solids are proportional to the cubes of their corresponding linear dimensions. The volumes of spheres are proportional to the cubes of their radii as the formula would indicate, but the radii are proportional to the diameters, also to the circumferences. The volumes of two spheres are proportional to the cubes of their diameters or the cubes of their circumferences. The same principle applies to other similar solids.

Summary

The volume of a rectangular solid = length × breadth × thickness.

The volume of a prism = area of end × length.

The volume of a cylinder = area of end × length.

The volume of a pyramid = area of base × $\frac{1}{3}$ perpendicular height.

The volume of a cone = area of base × $\frac{1}{3}$ perpendicular height.

The volume of a sphere = $\frac{4}{3}\pi r^3$.

The volume of a frustum of a pyramid = $\frac{1}{3}h(S^2+Ss+s^2)$.

The volume of a frustum of a cone = $\frac{1}{3}\pi h(R^2+Rr+r^2)$.

Exercise IXa

(1) How many $\frac{1}{4}$ inch cubes can be cut from a 2-inch cube?

(2) A cube, each of whose edges is .3 m., is formed of metal, 7 cubic centimetres of which weigh 50 grams. How many kilograms does the cube weigh?

(3) An ice-man delivered a block of ice 1 ft. 4 in. long, 10 in. wide, and 9 in. thick, and charged for 50 pounds. Assuming that water expands 10% in freezing, what was the shortage in weight?

(4) During a shower a rain gauge registered a fall of .75 inches. Find, in tons, the weight of water that fell on a quarter section of land.

(5) A school room accommodates 40 pupils and one teacher. For proper ventilation an adult requires 3000 cu. ft. of air per hour and a child $\frac{3}{4}$ of that amount. If air can be admitted at the rate of 5 ft. per second, what is the least size of opening necessary to allow sufficient air to enter?

(6) A tunnel 625 yards long and having a cross section of 64 square yards is excavated through rock of specific gravity 2.7. Find the weight of the rock removed.

(7) Water is poured into a tank 24 ft. 9 in. long, 9 ft. 4 in. wide, and 7 ft. 6 in. deep, at the rate of 12 gallons per second. Find the rate, in inches per minute, at which the water rises in the tank. How long will it take to fill the tank?

(8) The sides of the base of a triangular prism are 51 inches, 40 inches and 13 inches, its height is 18 inches;

(1) Find its volume.

(2) Find the edge of a cube which has the same volume.

(9) How many cubic yards of concrete are needed to make a semicircular walk in front of a school house, the walk to be 4 ft. wide, 44 ft. long on its shorter side, and the concrete to be laid 3 inches deep?

(10) A rectangular tent 10 ft. by 12 ft. has a three foot wall. The slant height of the roof is 6 ft. 3 in. Find, (1) the number of square yards of canvas used in making the tent (neglect seams), (2) the number of cubic feet of air space in the tent.

(11) A rectangular tank is 4.5 m. long, 3 m. wide and 2 m. deep (inside measurements). In what time could it be filled by two taps, one pouring in 6 litres of water in 4 seconds, the other 8 litres in 3 seconds?

(12) A rectangular vessel is 1.5 m. long, 72 cm. wide and 6 dm. deep (inside measurements). How many times will it contain the contents of a vessel holding 6 litres of water?

(13) If 36 lb. is the average weight of 1 cu. ft. of ensilage, and if a daily ration of 33 lb. is fed a cow for 180 days, what must be the capacity of a silo for a herd of 21 cows?

If the silo is cylindrical, with an internal diameter of 11 ft. 8 in., what must be the height?

(14) How many quarts will a hemispherical bowl hold if its internal diameter is 21 inches?

(15) The vertical height of a pyramid is 32 in. The base is a triangle whose sides are respectively 24 in., 21 in., and 28 in. long. Find the volume of this pyramid.

(16). The height of a cylinder is to the diameter of its base as 3 : 2. If the volume is $404\frac{1}{4}$ cu. in., what is its height?

(17) A spherical container having an internal diameter of 14 inches is filled with water. The water is then poured into a cylindrical vessel whose internal ~~radius~~ diameter is 14 inches. Find the depth of the water in the cylinder.

(18) A cylindrical measuring glass is graduated to indicate cubic centimetres. If the diameter of the glass is 2.5 cm., how far apart will the graduations be?

$$\left(\frac{1}{\pi} = .3183\right).$$

(19) The piston of a pump is 7 inches in diameter and makes a stroke of 18 inches. How many gallons of water will it deliver in an hour if it makes 30 double strokes per minute?

(20) A stone pyramid is 3 ft. 9 in. high, and its base is a rectangle 8 ft. long and 6 ft. wide. Find its volume. Find the total area of its four slant surfaces.

(21) A hollow spherical shell of iron is $1\frac{3}{4}$ inches thick and has an external diameter of 21 inches. Find the weight of iron in the shell, assuming the specific gravity of iron to be 7.7.

(22) Find the entire surface area of a circular cone which has an altitude of 12 inches and a volume of 616 cubic inches.

(23) How much ink will a fountain pen hold, the depth of the reservoir being 3.5 cm., and the internal diameter 7 mm.? How often can it be filled from a $\frac{1}{2}$-litre bottle?

(24) If air is .0013 times as heavy as water, find the weight, in kilograms, of the air in a hall 20 m. long, 15m. wide, and 8 m. high.

(25) How many tons of ice will an ice-house hold, that is 30 ft. long, 26 ft. wide, and 20 ft. high, if 2 ft. is allowed on all sides and above and below for sawdust? (Assume that water expands 10% in freezing).

Exercise IXb

(1) Fresh air enters a room through an opening 8 in. by 15 in., with a velocity of 6 ft. per second. How many cubic feet of air would enter the room in one hour?

(2) An engine boiler 4 ft. in diameter and 16 ft. long is traversed by 60 pipes each 3 inches in diameter, which convey the heat through the water. How much heating surface do these pipes apply to the water? How many gallons of water will the boiler hold?

(3) A solid sphere of lead 7 inches in diameter is melted and formed into one cylindrical plate 5 inches thick. Find the diameter of the plate.

(4) A farmer wishes to know how many tons of hay there are in a stack in the form of a cylinder 16 ft. in diameter and 7 ft. high, surmounted by a cone 8 ft. high. Allowing 512 cu. ft. to the ton, find the number of tons in the stack.

(5) A railroad fill 400 ft. long is 11 ft. wide at the top, 17 ft. wide at the bottom, and is 6 ft. deep. How many cubic yards of earth does it take to make the fill?

(6) To insure the ensilage keeping well, a silo should not be less than 35 ft. high. What must be its least diameter to hold enough ensilage to feed 44 cows 40 lb. each day for 148 days, if a cubic foot of ensilage weighs 37 lb.?

(7) The volume of a prism whose base is an equilateral triangle is 640 cu. in. If its height is 25 in., find the length of a side of the base.

(8) How many bushels, by measure, are there in a conical pile of grain 33 ft. in circumference and 4 ft. high?

(9) The radii of the ends of a frustum of a lead cone are 7 in. and 12 in. respectively, and the altitude is 9 in. Find its volume. How many lead balls of $\frac{1}{2}$ inch diameter can be made from this frustum?

(10) An iron pipe is 35 ft. long with inside diameter 6 in. and outside diameter 8 in. If the specific gravity of iron is 7.7, find, in pounds, the weight of the pipe.

(11) A cone, a hemisphere and a cylinder stand on the same base and have equal heights. Compare their volumes, and their total surface areas.

(12) A boiler is made in the form of a 6 foot cylinder $3\frac{1}{2}$ ft. in diameter, with hemispherical ends. How many gallons will it hold?

(13) The height of a cone is 16 in. and its volume is 1496 cu. in. Find the radius of the base.

(14) Two vessels, one cylindrical and the other a frustum of a cone, have the same depth, 9 inches. The diameter of the cylindrical vessel is 14 inches, and the diameters of the ends of the other vessel are 12 and 16 inches respectively. What is the ratio of their volumes?

(15) The radius of the base of a cone is 7 in. and its height is 16 in. Find the volume and the surface area of the frustum cut off at $\frac{3}{4}$ of the height from the base.

(16) Three solid iron cylinders have their heights in the ratio of 10 : 4 : 1, and their diameters in the ratio of 2 : 3 : 5. What is the ratio of their weights?

(17) A swimming-bath is 100 ft. long, 40 ft. wide, 8 ft. deep at one end and 4 ft. deep at the other. How many cubic feet of water will it hold?

(18) How many litres will a prismoidal vessel, 10 cm. high, contain if the base is a right angled isosceles triangle of side 10 cm.?

(19) Water flows through a circular opening in an irrigation dam at the rate of 10 miles per hour. If in 30 minutes sufficient water flows through to cover a quarter section of land to an average depth of $1\frac{3}{4}$ inches, find, in feet, the diameter of the opening.

(20) A lead pencil consists of a hollow cylinder of wood with a solid cylinder of graphite fitted into it. The diameter of the pencil is 7 mm., the diameter of the graphite 1 mm., and the length of the pencil 10 cm. Find, in grams, the weight of the pencil if the specific gravity of wood is 0.6 and the specific gravity of graphite 2.3.

(21) To make a cone-shaped tent, the altitude of which is 12 ft., and the diameter of the base 18 ft., how many square yards of canvas must be purchased, if 5% of it is used for seams, etc.?

(22) Find, to the nearest gallon, the number of gallons of water delivered per hour by a pump with a six foot stroke, and making 15 strokes per minute, the internal diameter of the barrel being 5 inches.

(23) A cistern 9 ft. long and 5 ft. wide is filled to a depth of 7.5 in. with pulp for making paper. How long a roll, 30 in. wide, can be made from this pulp, the thickness of the paper being 0.005 inches, supposing half the volume of the pulp to be lost in the process of drying?

(24) A coal bunker 4 ft. 9 in. wide at the top, 3 ft. wide at the bottom, and 10 ft. 6 in. deep. is 24 ft. long. Find the number of tons of coal it can hold, assuming that one ton occupies 42 cu. ft.

(25) What is the ratio of the circumference of two circles whose diameters are 2 in. and 4 in.? What is the ratio of the areas of two circles whose diameters are 2 in. and 4 in.? What is the ratio of the volumes of two spheres whose diameters are 2 in. and 4 in.?

(26) A sphere 4 cm. in diameter weighs 32 grams. What is the weight of a sphere made of the same material 8 cm. in diameter?

CHAPTER X

MEASUREMENTS

A. Surface Coverings

The carpeting of rooms with rugs of different standard sizes, leaving more or less margin around the outside, has largely done away with carpeting problems. It may be advisable, however, to completely cover the floor of a room with carpet. Oilcloth and linoleum are generally sold by the square yard and are manufactured in different widths, usually a whole number of yards. Other carpets are sold by the linear yard. The width is generally 27 inches. Whether the strips run crosswise or lengthwise on the floor may make a difference in the amount of waste.

To find the number of yards required to carpet a room divide the width of the room by the width of the carpet, if the strips run lengthwise, or the length of the room by the width of the carpet if they are to run crosswise. This gives the number of strips. Multiply the number of strips by the length of the strips required and express the result in yards. If the carpet has a pattern, waste may occur at the end of each strip (except the first) as an integral number of patterns must be cut off to have the pattern match. If part of a strip is required to complete a floor the whole strip must be purchased. For example, if a room requires $4\frac{1}{2}$ strips, 5 strips must be purchased.

The following is a working rule for finding the number of rolls of paper for the walls of a room:

Divide the perimeter of the room, less widths of doors and windows, by the width of the paper. This will give the number of strips. The height of the ceiling determines the length of the strips. If the paper has a pattern, waste may occur just as in carpeting. Multiply the length of each strip by the number of strips and express the total length in yards. Divide by 8, the number of yards in a roll, and this gives the number of rolls. To find the number of double rolls, divide by 16 instead of 8.

The border is sold by the linear yard. A length must be bought equal to the perimeter of the room.

In papering a ceiling the number and lengths of the strips may be found as in carpeting.

The methods of working problems dealing with building paper, paroid roofing, etc., are the same as those suggested for carpets and wall paper.

B. Timber

A board 1 foot long, 1 foot wide and 1 inch thick is a foot of lumber or board foot. Boards less than 1 inch thick are considered 1 inch thick in measuring. To find the number of feet of lumber in a board, plank, or stick of squared timber, multiply the length, in feet, by the width, in feet, by the thickness, in inches. In measuring boards the width is taken as the nearest whole number of inches.

Lumber is bought by the 1000 feet. $50 per thousand feet is quoted as $50 per M. Mouldings are bought by the linear foot. In calculating the number of feet of clapboards, shiplap, or matched lumber, to cover a given area, allowance must be made for the overlapping.

The number of board feet in a log is equal to the number of cubic inches in its volume divided by 144, or the number of cubic feet multiplied by 12.

The unit of roofing is the square, which contains 100 square feet. Theoretically a bundle of shingles contains 250 shingles 4 inches wide. Four bunches (1,000 shingles) when laid 4 inches to the weather will cover a square of roof. A shingle 4 inches to the weather covers 4 x 4, or 16, square inches. A square contains 14400 square inches. $14400 \div 16 = 900$, so 4 bunches to the square gives a liberal allowance for waste.

The pitch of a roof is the quotient of the width of the building by the vertical height of the ridge above the plate. If a building be 24 ft. wide and the ridge is 12 ft. above the plate, the roof has $\frac{1}{2}$ pitch. If the ridge is 8 ft. above the plate it has $\frac{1}{3}$ pitch. The length of the rafter can be calculated from the right angled triangle of which it is the hypotenuse allowing for the projection over the plate. Practically, the length is often determined by drawing the gable to a scale and measuring the hypotenuse of the right angled triangle.

C. Brick and Stone

Bricks vary in size, but 6 bricks, $8\frac{1}{2}'' \times 4\frac{1}{2}'' \times 2\frac{1}{2}''$, laid in the usual way will give a surface of 1 square foot, and a thickness of $4\frac{1}{2}$ inches. It will take 12 bricks to the square foot of surface for a 9 inch wall and a proportional number for other thicknesses.

Stonework is measured by the cubic yard or the perch. A perch contains $24\frac{3}{4}$ cu. ft., or for practical purposes 25 cu. ft. A cord of rough stone (128 cu. ft.) should lay 4 perches of wall.

D. Land

In the prairie provinces of western Canada land is laid out in *townships* 6 miles square. The townships in the row along the international boundary are all numbered 1. Those in the row immediately north of these are all numbered 2, etc.

A row of townships running north and south is a *range*. Ranges are numbered east and west from the principal meridian, which is situated a little west of Winnipeg. Meridian 102° is called the 2nd meridian, 106° the 3rd meridian, 110°, the Alberta-Saskatchewan boundary, is the 4th meridian, etc. Ranges are numbered west only from these. Owing to the convergence of the meridian lines as they go north, the ranges become fewer in number northward, the highest numbered range west of any meridian disappearing first.

31	32	33	34	35	36
30	29	28	27	26	25
19	20	21	22	23	24
18	17	16	15	14	13
7	8	9	10	11	12
6	5	4	3	2	1

A township is divided into 36 *sections*, each 1 mile square, numbered as in the diagram. Each section contains 640 acres, and may be divided into half sections, quarter sections, etc.

In eastern Canada farms are laid out in a less regular manner. A square $1\frac{1}{4}$ miles each way contains 1000 acres and is generally divided into 10 farms, each 80 rods by 200 rods and containing 100 acres. Blocks 1 mile wide and some multiple of 80 rods in length are divided into 80 acres farms.

Examples

(1) What will be the cost of carpet 27 inches wide, worth \$2.50 a yard, having an 18 inch pattern, to cover

the floor of a room 11 ft. by 14 ft., strips to run lengthwise?

No. of strips $= \frac{11}{2\frac{1}{4}} = \frac{44}{9}$ or 5 strips.

Length of first strip $= \frac{14}{1\frac{1}{2}}$ patterns $= 9\frac{1}{3}$ patterns, (or 14 ft.)

Length of other strips 10 patterns = 15 ft.

No of yards $\frac{14+4(15)}{3} = 24\frac{2}{3}$

Cost $\$2.50 \times 24\frac{2}{3} = \$61.67$.

(2) How many single rolls of paper 18 in. wide will be required for a room 9 ft. $\times$ $12\frac{1}{2}$ ft. with a 9 ft. ceiling? There are 2 doors each 3 ft. wide, and 1 window 5 ft. wide.

Perimeter $= 2(9+12\frac{1}{2}) =$ 43 ft.

Less 11 ft. for doors and window, 32 ft.

No. of strips $= \frac{32}{1\frac{1}{2}} = 21\frac{1}{3}$, 22 strips,

Total length $= \frac{22\times 9}{3}$ yards = 66 yards

No. of rolls $\frac{66}{8} = 8\frac{1}{4}$, 9 rolls.

(3) Find the cost of shiplap for a fence 140 ft. long and 6 ft. high at \$40 a thousand feet, if $\frac{1}{7}$ extra is allowed for overlap.

No. of ft. of lumber $= 140\times 6 = 840$

No. of ft. allowed for overlap $= 120$

Total No. of feet $= 960$

Cost $= \$40.00 \times \frac{960}{1000} = \$38.40$.

Exercise Xa

(1) How many square yards of oil cloth are needed for a floor 12 ft. by 20 ft. if 5 per cent of the material is wasted in matching?

(2) How many yards of carpet 30 in. wide will be required for a room 15 ft. long and 12 ft. wide, the carpet running lengthwise, and the pattern repeating every 6 inches?

(3) Find the cost, at 25 cents per square yard, of kalsomining the walls and ceiling of a room 16 ft. long, 12 ft. wide and 9 ft. high, deducting half the area of 4 windows each $3\frac{1}{2}$ ft. by 6 ft., and 2 doors, each 4 ft. by 7 ft.

(4) A congoleum rug is bought for a floor 15 ft. long and 12 ft. 6 in. wide. When put on the floor it leaves a space of 18 in. around the room uncovered. Find the cost of the congoleum at $1.50 per square yard.

(5) A room is 16 ft. long, 12 ft. wide and $8\frac{1}{2}$ ft. high. It has 3 windows each 3 ft. by 6 ft., and 2 doors each $3\frac{1}{2}$ ft. by 7 ft. Allowing for one half the area of the openings, find the cost of plastering the walls and ceiling at 50 cents per square yard.

(6) If it requires 1.16 barrels of cement, .44 cu. yd. of sand and .88 cu. yd. of gravel, to make a cubic yard of concrete, how much cement will it require to concrete the basement floor of a house 21 ft. by 18 ft. to a depth of 3 inches? At 50 cents per cubic yard find the cost of the sand and gravel used.

(7) At $25 per thousand, what will be the cost of the bricks for a building 32 ft. long, 24 ft. wide and 18 ft. high, the walls being 9 inches in thickness? In the walls there are 4 windows each $3\frac{1}{2}$ ft. by 6 ft., and 2 doors each

4 ft. by 8 ft. (Allow 7 bricks to the square foot for a $4\frac{1}{2}$ inch wall.)

(8) How many cords of stone are required for the walls of a basement 18 ft. wide, 24 ft. long and 9 ft. 6 in. high, the wall being 18 in. thick?

(9) What will it cost for the material to paint (two coats) a house 24 ft. wide, 30 ft. long and 12 ft. from the foundation to the plates? It has two gables each 24 ft. wide and 8 ft. high. The paint costs $3.50 per gallon, and one gallon covers 250 square feet with two coats.

(10) A room is 18 ft. long, 12 ft. wide and 9 ft. high, and has 3 windows each 4 ft. by $6\frac{1}{2}$ ft., and 2 doors each 4 ft. by 7 ft. Find the cost of paper, 18 in. wide, to paper the walls and ceiling, the wall paper costing 80 cents per double roll, and the ceiling paper 20 cents per single roll.

(11) A room is 12 ft. long and 10 ft. wide. What will carpet, 24 inches wide with a pattern repeating every 9 inches, cost for this room at $1.20 per yard?

(12) A room is 18 ft. long, 12 ft. wide and 10 ft. high. What will it cost to kalsomine the walls and ceiling at 20 cents per square yard, making deduction for 3 windows each $3\frac{1}{2}$ ft. by 6 ft., 2 doors each 4 ft. by 7 ft., and a 9 inch baseboard?

Exercise Xb

(1) Find the cost of 30 scantlings 2 in. by 4 in., and 20 ft. long at $40 per M.

(2) How many feet of lumber are there in a piece of timber 16 inches square and 21 ft. long?

(3) A bridge 50 ft. long and 24 ft. wide is covered with 2 inch planks. What will the planking cost at $45 per M?

(4) A sidewalk, $\frac{1}{2}$ mile long and 6 ft. wide, is made of 2-inch plank. The planks rest on two lines of scantling

2 in by 4 in. If the cost of the planking is \$40 per M and the cost of the scantling \$45 per M, find the total cost of the material for the sidewalk.

(5) How many feet of 2-inch lumber are there on a car 38 ft. long and 8 ft. wide if the lumber is piled 6 ft. high?

(6) Find the amount of the following bill:

15 pieces, No. 1 Fir, 2″×6″, and 14′ long, at \$60 per M.

20 pieces, No. 1 Cedar, 4″×4″, and 16′ long, at \$45 per M.

12 pieces, No. 1 Oak, 2″×8″, and 12′ long, at \$250 per M.

10 pieces, No. 1 Maple, 2″×4″, and 12′ long, at \$200 per M.

(7) At \$35 per M, what will it cost to lay a floor in a shed 16 ft. wide and 30 ft. long, with 2-inch plank?

(8) A close board fence, 6 ft. high, is built around a block of land 400 ft. long and 240 ft. wide. The inch boards are nailed to two lines of scantling, 2 in. by 4 in. The scantling is nailed to posts placed 10 ft. apart. If the posts cost 40 cents each and the other lumber \$30 per M, find the total cost of the material in the fence.

(9) How much inch lumber will be required to make 20 covered boxes each 4 ft. 6 in. long, 3 ft. 4 in. deep and 2 ft. 8 in. wide?

(10) A coal bin 10 ft. long, 8 ft. wide and 6 ft. high, without a floor, is made of inch lumber nailed to 20 upright scantlings 2 in. by 4 in. These pieces of scantling rest on base scantling 4 in. by 4 in. At \$55 per M, what will the lumber cost, allowing nothing for waste?

(11) The width of the building is 24 ft. and the rise of the roof is 9 ft. Find the length of a rafter for the gable roof if it projects 18 inches over the side of the building.

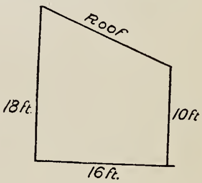
(12) A barn 47 ft. long, 32 ft. wide and 24 ft. high (from foundation to plate) has a gable roof with a rise of 12 ft. The rafters are 2 in. by 6 in. and project 18 inches over the side wall. The roof projects 18 inches over the end walls.

(a) If the rafters are placed 2 ft. apart from centre to centre, find their cost at $40 per M. (Allow for waste to make length an even number of feet).

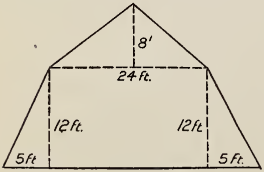
(b) At $35 per M, find the cost of inch lumber required to enclose this building, including the sheeting for the roof.

(c) Find the cost of shingles for the roof at $6 per thousand, if they are laid 4 inches to the weather.

(13) The accompanying figure represents the end of a shed 30 ft. long, and other dimensions as shown. At $30 per M, what will be the cost of inch lumber required to enclose the building?



(14) The figure shows part of the end of a barn 57 ft. long, with a curb roof and with other dimensions as shown. The rafters project 18 inches over the side walls and the roof projects 18 inches over the end walls. At $6 per thousand, what will the shingles cost for the roof of this building if laid 4 inches to the weather?



(15) A living room floor 16 ft. long and 12 ft. 6 in. wide is laid with maple $2\frac{1}{2}$ in. wide. At $175 per M, what would the flooring cost if $\frac{5}{16}''$ extra is required on each board for tongue and groove and 5% is allowed for squaring?

(16) How many slates 6 in. wide and laid 8 in. to the weather are required for the gable roof of a building 24 ft. wide, the rise of the roof being 9 ft., the ridgepole 42 ft. long, and the rafters projecting 12 inches over the side walls?

(17) A farmer owns the northeast quarter of section 16 and the east half of section 22. Draw a township plan and locate his land. How many acres of land does he own? How many rods of fencing would be required to enclose his land? If the district school is located on the southwest corner of section 28 and his home is at the northeast corner of section 16, how far will his children have to travel to school?

(18) A table is built according to the following specifications:

Top—4 ft. by $3\frac{1}{2}$ ft. and 2 in. thick.

4 legs—2 ft. 4 in. long and 3 in. square.

4 pieces mortised into the legs—

2 pieces 28 in. long, 5 in. wide and 1 in. thick

2 pieces 21 in. long, 5 in. wide and 1 in. thick

Find the cost of the lumber at 45 cents per board foot.

REVIEW EXERCISE I—MENSURATION

(1) A military tent is 12 ft. high. Its shape is that of a cone standing on a cylinder, the diameter of which is 14 ft. and height 6 ft. If $\frac{1}{12}$ of the canvas used in making this tent is allowed for seams and waste, find its cost at 45 cents per square yard.

(2) A farmer makes a silo which is in the form of a vertical hollow cylinder with concrete walls and floor. The inner diameter of the silo is 12 ft. and inside height 30 ft. The walls and floor have a uniform thickness of 14 inches. How many cubic yards of concrete will be required to make the silo?

(3) A rectangle, a semicircle, and an isosceles triangle have equal bases and equal altitudes. Show that the volumes generated by revolving them about their bases are as 3 : 2 : 1.

(4) A canal is 8 ft. deep, 18 ft. wide at the surface, and 12 ft. wide at the bottom. Find the number of gallons of water that pass any point in an hour when the water is flowing at 2 miles per hour.

(5) A railway cutting 10 metres deep has to be made with one side vertical and the other inclined at 45° to the vertical. The bottom is to be 12.4 metres wide. How many steres of earth and rock must be removed per kilometre?

(6) What must be the diameter of a cylindrical vessel to hold twice as much as one with a diameter of 20 inches, if their heights are the same?

(7) What is the weight of a hollow steel pillar 20 ft. long whose external diameter is 5 in. and internal diameter $3\frac{1}{2}$ in., if the specific gravity of steel is 7.5? What is the diameter of a solid pillar of the same weight and height?

(8) A circular grindstone when new has a diameter of $3\frac{1}{2}$ ft. and a $3\frac{1}{2}$ inch face. How much has it lost in weight when the diameter is worn down 7 inches, assuming the specific gravity of sandstone to be 2.4?

(9) A cylindrical tree trunk 10 ft. long and 14 in. in diameter is planed down so as to form the largest possible prism whose cross section is an equilateral triangle. Find the number of cubic feet in the prism.

(10) Water flows in a V-shaped gutter. Find the number of cubic metres discharged in a day if the depth is 20 cm. and the speed of the water a metre per second, the sides of the gutter meeting at right angles.

(11) Find the weight in pounds of an iron shaft 13 ft. 6 in., high, with rectangular ends, the dimensions of one end being 10.5 in. by 18 in., and the other 7 in. by 12 in. (Assume that the specific gravity of iron is 7.7.)

(12) If 1550 square inches equal 1 square metre, find, to two decimal places, the number of acres in a hectare.

(13) A farmer decides to build a barn 48 ft. long, 24 ft. wide, with walls 20 ft. high. The peak of the roof is to be 9 ft. above the plates. The roof is to project one foot over the side walls and one foot over the end walls.

(1) At $35 per M, what will be the cost of the lumber to close in the walls and gable ends?

(2) Find the cost of sheeting for the roof at $25 per M.

(3) If shingles cost $3.50 per bundle and it takes 4 bundles to cover 100 sq. ft., find the cost of the shingles for the roof.

(14) Two wheels fixed on parallel shafts 18 ft. apart revolve in the same plane. The radii of the two wheels

are respectively 3 ft. and 6 ft. Find the length of belting required to pass around the wheels if the belting is to cross itself between the wheels.

(15) A stretch of 10 miles of a river of an average width of 120 yards rose 5 ft. 5 in. in consequence of a heavy rainfall.

(1) How many million gallons of additional water in this stretch of the river did this represent?

(2) If the area on which the rain fell was 30 square miles per mile length of the river, and the rainfall was 5.2 inches, find what fraction of the total rainfall was represented by the additional water in this stretch of the river.

(16) A cylindrical vessel whose diameter is 70 mm., (inside measurement) contains 3 litres of water when full. Find the depth of the vessel expressed in centimetres.

(17) A rectangular vessel which is 3 m. long and 25 dm. wide (inside measurements) contains water to a depth of 8.5 cm. Find:

(1) The area of the surface of the water in square metres.

(2) The number of litres of water in the vessel.

(3) The weight of the water in kilograms.

(18) Three equal circles touch one another at the points A, B and C. If the diameter of each circle is 14 in., find the area enclosed by the three arcs AB, BC and CA.

(19) Prove that the area of a square is half the square on its diagonal.

(20) The top of the roof of a house is flat and is 4 ft. square. The roof slopes down equally on all four sides to the eaves which form a square 20 ft. to the side. The

vertical distance from the level of the eaves to the top is 6 ft. How many bundles of shingles, each covering 25 sq. ft., will be required for the sides of the roof?

(21) A motor track consists of two straight parallel sides each 286 yards long and two semicircular ends. What must be the distance between the sides, if it is just half a mile around the inside of the track?

(22) If a ton of coal on board a steamer occupies 40 cu. ft. of space, and if the specific gravity of a lump of coal is 1.20, find the fractional part of the space that is lost in the bunker owing to the irregular shapes of the lumps of coal.

(23) A pile of grain in the shape of a cone is 12 ft. in diameter at the bottom and the slant height is 10 ft. Find the number of bushels, by measure, in the pile.

(24) If the specific gravity of ivory is 1.82, find the weight of a set of 4 billiard balls 2 inches in diameter.

(25) Iron is 10 times as heavy as wood. An iron cylinder is 3 times as high as a wooden one, but the diameter of the wooden one is one and one-half times that of the iron one. In what ratio are their weights?

(26) A closed cylindrical tin, capable of containing a given volume, requires least material for its construction when the height of the tin is equal to the diameter of its circular base. Express the height of such a tin if the volume contained is 1 cubic foot.

(27) A clothes boiler has semicircular ends. How many gallons of water will a boiler contain whose extreme length is 30 in., height 16 in., and width 14 in.?

(28) What is the area of the longest circular running track 7 ft. wide that can be laid out in a lot 196 ft. square? ($\pi = 22/7$).

(29) The largest possible cone is turned in a lathe out of a cylinder 6 in. long and 3 in. in diameter. What part of the cylinder goes into shavings?

(30) A rain gauge consists of a cylindrical receiving vessel of diameter 4 in., surmounted by a funnel with a diameter of 14 in. at the top. After a rainfall the depth of water in the receiver was 5 in. What was the precipitation?

(31) A haymow in a barn is 32 ft. wide, 40 ft. long and 15 ft. high to the eaves of the roof. The highest point of the roof is 12 ft. above the level of the eaves. If the mow is completely filled, how many tons does it contain assuming that one ton occupies approximately 450 cubic feet?

(32) From a given strip of land with a firm surface, 5.34 inches of water evaporated during a given time. From an adjacent strip of like size and soil, the surface of which was kept well cultivated to a depth of 3 inches, only .88 inch evaporated in the same time. How many tons of water were saved the soil by cultivating 80 acres?

(33) The cross section of an iron wire is 1 sq. mm. A cubic centimetre of the wire weighs 7.9 g., and the force required to break the wire is the weight of 35 Kg. What length of the wire suspended by one end will just break under its own weight?

(34) A block of wood is 10 cm. by 8 cm. by 5 cm., and its relative density is .45. Find the weight of the wood. A cavity is made in the block and 250 grams of lead inserted. What must be the size of the cavity if the combined weights of the remaining wood and the lead would allow the block just to float in water?

(35) In a semicircle ABCD of diameter 5 ft. 10 in., a rectangle FBCE is inscribed so that AF is equal to 1 ft. 3 in. Find the area of the rectangle.

(36) A bucket has the shape of a frustum of a cone. The diameters of its upper and lower interior sections are 9 in. and 8 in. respectively. The depth of the bucket is $9\frac{1}{2}$ in. Find (a) what decimal of a cubic foot of water it will hold when full, (b) the number of gallons. (c) the weight of the water. ($\pi = 3.1416$).

(37) The cross section of some uniform wire is 3 sq. mm., and its length is 4.2 m. It is heated and drawn out into a uniform wire 25 m. long. What is now the area of the cross section, to the nearest hundredth of a square millimetre?

(38) In the middle of a rectangular plot of ground 88 yd. by 70 yd., is dug a circular tank of radius 14 yd., and 10 yd. deep. If the earth taken out be spread evenly over the rest of the plot, find by how much the rest of the plot is raised.

(39) A boiler is cylindrical in shape with ends which are hemispheres. If the internal diameter of the boiler is 5 ft. and its length 12 ft., find the volume of water which it can hold?

(40) The surface of the seat of a swing is $10\frac{1}{2}$ ft. from the beam from which it is suspended. It swings through an angle of 32° on each side of the vertical. What is the length of the arc traced out by a point on the surface of the seat?

(41) A wholesaler finds that he saves appreciably in the cost of packing material if he makes the 1 lb. packets of tea cubical rather than cuboidal. What fraction of the material is saved when a cubical form is adopted, in place of a cuboid, whose edges are in the ratio of 4 : 2 : 1?

(42) If 1 inch and 9 feet represent the relative diameters of the earth and the sun:

(a) Compare the areas of their surfaces.

(b) Compare their volumes.

REVIEW EXERCISE II—MENSURATION

(1) Find the distance from a lower corner to the opposite upper corner of a rectangular room 32 ft. long, 24 ft. wide and 16 ft. high.

(2) An ice dealer stored the ice from 4 acres of a lake, the average thickness of the ice being 2 ft. Find the weight in tons of the ice stored, assuming that water expands 10% in freezing.

(3) A lot was bounded as follows: beginning at the northeast corner the line ran west 40 ft., then south 80 ft., then east 120 ft., and then to the northeast corner. Find the cost of sodding this lot at 25 cents per square yard.

(4) The volume of a cylinder is given by the formula $V = \pi r^2 h$, when $V =$ number of cubic units in the volume, $r =$ number of linear units in the radius, and $h =$ number of linear units in the height. If $r = 17.82$ in., and $h = 12.95$ in., each expressing the number of inches to the nearest hundredth, find, to the nearest hundredth, the number of cubic inches in the volume. ($\pi = 3.1416$).

(5) 1 metre = 39.37 inches, to the nearest hundredth of an inch. How many square inches are there in a square metre, to the nearest hundredth of a square inch.

(6) The dimensions of a box, to the nearest tenth of an inch, are 15.8 in., 11.9 in., and 6.7 in. Find the volume, to the nearest tenth of a cubic inch.

(7) The sides of a rectangle are to each other as 4 is to 3. The diagonal is 55 inches. Find the area.

(8) A swimming bath 60 ft. long and 20 ft. wide, is 3 ft. 6 in. deep at the shallowest end and 6 ft. 6 in. deep at the deepest end. The depth increases uniformly from one end to the other. How many cubic feet of water does it contain? Find the area of the floor of the bath.

(9) Walking at the rate of 3 miles an hour a man walks round a rectangular field of 24 acres in 16 minutes. Find the dimensions of the field.

(10) A fence around a square field costs $180 at 75 cents per rod. What would have been the cost if the field had been in the form of a rectangle whose sides were in the ratio of 4 : 1?

(11) A cistern is 6 ft. in diameter. How much will a one-inch rainfall upon a roof that is 60 ft. by 40 ft. raise the level of the water in the cistern?

(12) To irrigate a 20-acre field, water is run full bore through a 6-inch pipe with a velocity of one foot per second. How long will it take to deliver one inch of water over the entire field?

(13) Water enters a tank through two pipes having diameters of $\frac{3}{4}$ inch and $1\frac{1}{4}$ inches respectively. Find the diameter of the waste pipe that will allow the water to run out as fast as it runs in, running at the same velocity.

(14) A cylindrical beaker 10 cm. in height inside just holds a litre of water. Find the inside diameter correct to the nearest millimetre.

(15) The hole in a golf green is $4\frac{1}{2}$ inches in diameter. If the area of the hole were doubled, what would the diameter be?

(16) A closed cylindrical vessel has its height equal to the diameter of its circular base. Find the height of such a vessel if the volume contained is 1 quart. ($\pi = 3.14159$).

(17) A pint of water weighs $1\frac{1}{4}$ lb. One kilogram = 2.206 lb. One cubic centimetre of water weighs 1 gram. Find, to the nearest hundredth, the number of pints in a litre.

(18) Forty-eight tins of canned fruit are packed into and just fit a rectangular box. Each tin is cylindrical in shape, the diameter being 3.5 in. and the depth 5.6 in. The tins are arranged in layers of a dozen each with the circular ends horizontal. Find, to the nearest cubic inch, the volume of the vacant space in the box.

(19) Shrapnel bullets are of lead, spherical, and of $\frac{1}{2}$ inch diameter. Calculate their weight per thousand if 1 cubic inch of lead weighs .41 lb.

(20) A wooden cone for demonstration purposes has a diameter of 8 in. and is 18 in. high. Calculate its weight if it is made of pine weighing .0318 lb. per cu. in.

(21) It is desired to cut down the cone mentioned in the previous example, into a square pyramid, keeping the diameter of the cone as the diagonal of the square base. What will be the volume of the square pyramid?

(22) A circle and a square have equal areas. Find the ratio of their perimeters.

(23) A triangle has sides 13 in., 14 in., and 15 in., respectively. Find the area of a triangle similar to this, whose side corresponding to the 14 in. side is 21 in.

CHAPTER XI

PERCENTAGE AND SIMPLE APPLICATIONS OF PERCENTAGE

A. Percentage

The Latin words "per centum" are in common usage abbreviated to "per cent." These may be further shortened to "p.c." and finally to "%". "Per cent" means "per hundred" or "on the hundred." The majority of business transactions involve percentage in some way.

5% means 5 parts out of 100 or $\frac{5}{100}$ or .05.

$12\frac{1}{2}$% means $12\frac{1}{2}$ parts out of 100 or $\frac{1}{8}$ or .125.

5% of \$32 means $\frac{5}{100}$ of \$32 = \$1.60.

To express any given per cent as a fraction it is simply necessary to write 100 under the number expressing the per cent.

The whole of anything is 100% of it.

$\frac{1}{2}$ of anything is $\frac{1}{2}$ of 100% of it. This is equivalent to $(\frac{1}{2}\times 100)\%$, or 50% of it.

$\frac{1}{5}$ of anything is $(\frac{1}{5}\times 100)\%$, or 20% of it.

.28 of anything is $(.28\times 100)\%$, or 28% of it.

To express a fraction, vulgar or decimal, as a percentage, multiply it by 100.

Examples

(1) 8 is what % of 440?

8 is $\frac{8}{440}$ of this number. $\frac{8}{440} = (\frac{8}{440}\times 100)\% = 1\frac{9}{11}\%$

Then 8 is $1\frac{9}{11}\%$ of 440.

(2) \$42 is 7% of what sum?

Let the sum be \$x.

$\frac{7}{100}x = 42$

$x = \frac{42 \times 100}{7} = 600.$ The sum is \$600.

(3) What number increased by 40% of itself is equal to 98?

When 40% of the number has been added the result will be 140% of the number.

140% of the number = 98

100% of the number $= \frac{9800}{140} = 70.$

The number is 70.

B. Profit and Loss

One of the commonest applications of percentage is the expression of profit or loss in a business transaction. The percentage of profit or loss is generally calculated upon the cost price of the commodity. This should be the gross cost, which includes the original, or prime cost together with other expenses incidental to the transaction. Profit is sometimes expressed as a percentage of the proceeds.

Examples

(1) What is the gain or loss per cent on cost when hats cost \$54 a dozen and are sold at \$6.50 each?

Selling Price of 1 hat	\$6.50
Cost Price of 1 hat	4.50
Gain	\$2.00

Gain per cent on cost $= (\frac{200}{450} \times 100)\% = 44\frac{4}{9}\%.$

(2) A firm buys chairs at \$15 a dozen and sells them so as to gain $37\frac{1}{2}\%$ of the proceeds. What does one chair sell for?

$37\frac{1}{2}\%$ of the proceeds is gain,

$62\frac{1}{2}\%$ of the proceeds pays the cost,

$62\frac{1}{2}\%$ of the selling price of 12 chairs = \$15,

100% of the selling price of 12 chairs $= \dfrac{\$1500}{62\frac{1}{2}} =$ \$24.

1 chair sells for \$2 00.

C. Trade Discount

Manufacturers and wholesalers issue catalogues of their wares with fixed price lists. The prices shown are often considerably above what are actually charged. The actual price charged is the list price, less a certain percentage of this list price. Changes in prices then do not necessitate the preparation of new price lists of the goods but simply a change in the discount or discounts offered.

An article listed at \$12.00 is bought for \$9.00 if the discount is 25%, thus:

List price	\$12.00
Discount 25%	3.00
Buying price	\$ 9.00

An increase in the price of the article would be indicated by a decrease in the discount off the list price, perhaps to 20%, and the article listed at \$12.00 would cost \$9.60. When the price comes down, instead of increasing the trade discount to 30% or 35% or some

other rate the decrease in price may be quoted as a second discount. A second discount may also be allowed on orders totalling a stated amount or over. The article listed at $12.00 might be quoted at 25% and 20% off the list price, thus:

List price	$12.00
Discount 25%	3.00
Remainder	$9.00
Discount 20% (of $9)	1.80
Buying price	$7.20

To find the single discount to which these two discounts are equivalent:

The total discount allowed is $3.00+$1.80=$4.80.

The single discount would be $\frac{4.80}{12.00} \times 100\% = 40\%$

Often three, or sometimes more, discounts are quoted, the last being generally a discount for cash.

Example

What is the cash price of an article listed at $12.00 with discounts 25% and 20% off? Terms, 60 days or 2% discount for cash.

List Price	$12.00	
Discounts as above	4.80	
	$7.20	
Cash Discount	.14	(nearest cent)
Net Price	$7.06	

A retail dealer may give a discount either for cash or in case of special sale. This is generally a single discount off the marked price.

D. Commission

Commission merchants receive consignments of produce which they sell for the shipper or consignor, or, acting as agents, buy goods and send them to their employers. The money the commission merchant or agent is paid for his services is called commission. It is sometimes calculated as so much per unit of measurement or weight as $1\frac{1}{2}$ cents a bushel for grain, or 50 cents a ton for hay. It is more generally a percentage of the price of the goods.

It is necessary to remember that the selling commission is a certain percentage of the selling price and the buying commission is a percentage of the buying price.

Examples

(1) A commission merchant sells a consignment of 240 dozen eggs at 28 cents a dozen on a commission of 4%. What is his commission and how much money does he send back?

240 dozen eggs at 28¢	$67.20
Commission, 4%	2.69
Amount remitted	$64.51

(2) An agent received $1,470 to invest in sugar. He buys at $10.00 a cwt. on a commission of 5%. What is his commission? How much sugar does he buy?

When he buys a dollar's worth of sugar he pays \$1.00 for the sugar and takes 5 cents for himself as commission, thus using \$1.05 of his employer's money. The commission is thus .05 out of 1.05, or $\frac{5}{105}$ of the money received.

Commission,	$\frac{5}{105}$ of \$1,470	\$ 70.00
Sugar costs	$\frac{100}{105}$ of \$1,470	1400.00
	Total	\$1470.00

Commission \$70. Sugar bought $\frac{1400}{10}$ cwt. = 140 cwt.

Buying commission is sometimes complicated by instructions to pay freight in advance, requiring the agent to take purchase price, commission and freight out of the money sent. It is advisable in this case to consider a definite amount of merchandise. Add together the prime cost, commission and freight. This total will pay all charges on the amount of merchandise under consideration and the amount that can be bought with the sum sent can be calculated.

Example

An agent is sent \$7,758. He is to buy tea at $62\frac{1}{2}$ cents a pound, retaining his commission of 2%, and pay freight in advance at 90 cents a cwt. What is his commission, and how much tea does he buy?

1 cwt. of tea costs	\$62.50
Commission, 2% of \$62.50	1.25
Freight	.90
Total Cost	\$64.65

$$\text{No. cwt. bought} = \frac{7758.00}{64.65} = 120.$$

120 cwt. tea @ \$62.50	\$7500.
Commission, 2% of \$7500.	150.
Freight @ 90¢	108.
Check	\$7758.

He buys 120 cwt. of tea and receives a commission of \$150.

Had the agent received \$7,775 he probably would have bought the same amount of tea and sent back \$17.

Sometimes goods are sent to an agent to be sold on commission and the proceeds are to be re-invested in other goods, also on commission. We have in this case two commissions or "double commission" to deal with.

Example

An agent sold 4000 bushels of wheat at \$1.53 a bushel on a commission of 3% and invested the proceeds, less a commission of 2%, in flour at \$4 a cwt. What was his total commission and how much flour did he buy?

4000 bushels wheat @ \$1.53	\$6120.00
Selling commission of 3%	183.60
Balance	\$5936.40
Buying com. $\frac{2}{102}$ of \$5936.40	116.40
Invested in flour	\$5820.00

Amount of flour bought $\frac{5820}{4}$ cwt. = 1455 cwt.

Total commission	\$183.60
	\$116.40
	\$300.00

Check. Total commission should be $\frac{3+2}{100+2}=\frac{5}{102}$ of selling price.

$\frac{5}{102}$ of \$6120 = \$300, which verifies the result.

This check is worth remembering.

Note: If the selling price be \$x, the selling commission at the rate of a % and the buying commission b%, the total commission will be $\frac{a+b}{100+b}$ of \$x.

This can be established algebraically.

Suppose the selling price is \$x.

$$\text{Selling commission} = \frac{a}{100} \text{ of } \$x = \$\frac{ax.}{100}$$

$$\text{Balance} = \$x - \frac{\$ax}{100} = \$\frac{x(100-a)}{100}$$

$$\text{Buying commission} = \frac{b}{100+b} \text{ of } \$\frac{x(100-a)}{100} = \$\frac{bx(100-a)}{100(100+b)}$$

$$\text{Total commission} = \$\frac{ax}{100} + \$\frac{bx(100-a)}{100(100+b)} = \$\frac{100x(a+b)}{100(100+b)}$$

$$= \$\frac{x(a+b)}{(100+b)} = \frac{a+b}{100+b} \text{ of } \$x.$$

The following table of equivalent fractional forms should be learned:

Percentage Fraction		*Common Fraction*		*Decimal Fraction*
50%	=	$\frac{1}{2}$	=	$.5$
25%	=	$\frac{1}{4}$	=	$.25$
20%	=	$\frac{1}{5}$	=	$.2$
10%	=	$\frac{1}{10}$	=	$.1$
75%	=	$\frac{3}{4}$	=	$.75$
$12\frac{1}{2}\%$	=	$\frac{1}{8}$	=	$.125$
$33\frac{1}{3}\%$	=	$\frac{1}{3}$	=	$.\dot{3}$
$66\frac{2}{3}\%$	=	$\frac{2}{3}$	=	$.\dot{6}$
$16\frac{2}{3}\%$	=	$\frac{1}{6}$	=	$.1\dot{6}$
$6\frac{1}{4}\%$	=	$\frac{1}{16}$	=	$.0625$
$37\frac{1}{2}\%$	=	$\frac{3}{8}$	=	$.375$
$62\frac{1}{2}\%$	=	$\frac{5}{8}$	=	$.625$
$14\frac{2}{7}\%$	=	$\frac{1}{7}$		

Exercise XIa

(1 to 17 oral)

1. Express as common fractions reduced to lowest terms, the following percentage fractions:

$28\frac{4}{7}\%$, $47\frac{1}{2}\%$, $22\frac{1}{2}\%$, $83\frac{1}{3}\%$, $11\frac{1}{4}\%$, $8\frac{1}{3}\%$, $2\frac{1}{2}\%$, $7\frac{1}{2}\%$, 125%, 225%, 175%, $\frac{1}{2}\%$, $\frac{3}{4}\%$, $\frac{4}{5}\%$, $1\frac{1}{5}\%$.

(2) Express as decimal fractions the following percentage fractions:

8%, 16%, $12\frac{1}{2}\%$, $6\frac{3}{4}\%$, $7\frac{1}{2}\%$, $.6\%$, $.05\%$, $\frac{1}{5}\%$, $\frac{1}{8}\%$, $\frac{5}{8}\%$, 3.75%, 8.25%, $.375\%$, $25\frac{1}{4}\%$.

3. Express as percentage fractions and decimal fractions each of the following common fractions:

$\frac{3}{20}$, $\frac{9}{10}$, $\frac{1}{8}$, $\frac{1}{11}$, $\frac{2}{3}$, $\frac{3}{25}$, $\frac{5}{12}$, $\frac{8}{9}$, $\frac{2}{75}$, $\frac{1}{16}$, $\frac{7}{8}$, $\frac{3}{200}$, $\frac{7}{15}$.

(4) A hat that cost \$3.60 was sold at an advance of $33\frac{1}{3}\%$. What was the selling price?

(5) A piano was sold for \$400. This was a loss of 20% of the cost price. Find the cost price.

(6) A man withdrew \$90 from the bank. This sum was 60% of what he had on deposit. What was the full amount of his deposit?

(7) A merchant sold a coat for \$35 and thereby lost $12\frac{1}{2}\%$. What did the coat cost him?

(8) What per cent is 6 of 30? 15 of 90? 21 of 84? 27 of 81? 8 of 20? 42 of 63? 27 of 45?

(9) (a) 5% of? $=30$ (b) $12\frac{1}{2}\%$ of $?=20$
(c) $33\frac{1}{3}\%$ of $?=71$ (d) $16\frac{2}{3}\%$ of $?=50$
(e) $37\frac{1}{2}\%$ of $?=12$ (f) $3\frac{1}{3}\%$ of $?=15$
(g) $9\frac{1}{11}\%$ of $?=30$

(10) If in a rural school there are 12 boys and 18 girls, what per cent. of the enrolment are boys? girls?

(11) A man asked \$180 for a horse. If he accepted \$160, what per cent less than the price asked did he accept?

(12) What per cent profit does a grocer make who buys seed at 80 cents per bushel and retails it at 3 cents per quart?

(13) During the month of June a farmer sells 120 gallons of milk; during the month of July he sells 150 gallons. What is the per cent. of increase?

(14) A bankrupt's assets are \$8,000 and his liabilities are \$12,000. He owes W. Jones \$300. What per cent. of his debts can he pay? How much will Jones receive?

(15) I have money invested in a mortgage which pays 8%. My annual income from this investment is $160. How much have I invested?

(16) I bought a lot for $360. During the first year it increased in value $16\frac{2}{3}$%. During the second year its value decreased $16\frac{2}{3}$%. How much was it then worth?

(17) After spending 72% of my year's salary I have saved $560. What is my year's salary?

(18) A merchant bought 2 cwt. of tea that cost him 45¢ per lb. He sold 120 lbs. at 70¢ per lb., 60 lbs. at 50¢ per lb., and the balance at 30¢ per lb. Find his per cent of gain.

(19) A ship consumes 1121 tons of coal during a certain voyage. On her next trip she encountered heavy weather and so burnt 1427 tons. What is the increase per cent of the coal consumption?

(20) Bronze is made of copper, tin, and zinc, the percentages by weight of the two former being 90.5 and 8.4 How many pounds of bronze would be made if one pound of zinc were used?

(21) An iron rod is "turned down" in a lathe so as to reduce its weight by 12%. It now weighs 22 lbs. What was its original weight?

(22) A man whose yearly salary is $3000 saves $1200 after paying rent and living expenses. Rents advance 20% and living expenses 25% and he now saves only $768. Find the sum now paid for rent.

(23) A cube has an edge 1 ft. 8 in. long. If the length of the edge were diminished by 5% what would be the percentage reduction in the surface? In the volume?

(24) A plan is drawn to the scale of 3 cm. to the foot and is to be reduced in scale to 1 inch to the foot. What reduction per cent. will there be in (1) lengths of lines on the plan, (2) the area of the plan? (1 m. = 40 inches, approximately).

(25) 17% of a certain number, together with 7% of three times the number equals 76. Find the number.

(26) What per cent. profit is equivalent to using a false balance which weighs only 14 oz. to the pound?

(27) 40% of the candidates presenting themselves for examination were girls. 25% of the girls and 15% of the boys failed. What percentage of the number of candidates was successful?

(28) The population of a certain city is now 185,120. Four years ago its population was 160,000. Find its population four years hence, supposing the rate per cent. of increase is the same.

(29) If two gallons of hot copper sulphate solution containing 70% of copper sulphate are mixed with 1 gallon containing 40%, what is the percentage of copper sulphate in the solution?

(30) A dealer professes to sell sugar at cost price, but uses fraudulent weights, thereby making a profit of 12%. What does his pound weight really weigh?

(31) Sea-water contains $2\frac{1}{2}$% of salt. What weight of water would be required to yield half a ton of salt?

(32) The entrance fee to an exhibition being reduced by one-quarter, the daily attendance is increased by 30%. What is the percentage decrease in the daily receipts?

(33) If a decrease of 30% in the duty on tobacco led to an increase of 40% in the amount imported, what would be the percentage increase or decrease in the revenue derived from this source?

(34) A man held 25 per cent. interest in an estate and transferred 25 per cent. of his interest to another man for $5,250. Find the estimated value of the estate.

(35) If the length of an iron rail increases by .00065% for every degree F. through which it is heated, find:

(a) The increase in length of 100 miles of iron rails between temperatures of 40° and 90° F. (nearest yard).

(b) The amount of play that must be allowed at one end of an iron bridge 750 feet long, if the other end is fixed and the temperature varies from −20° F. to 100° F. (answer to the nearest tenth of an inch).

(36) Water in freezing expands in volume by 10%; by how much per cent. does ice contract on melting.?

(37) In the erection of a bridge, five times as much was paid for building material as for labor. Had 19% less been expended for material and 15% more for wages, it would have cost $4,680. What was its actual cost?

Exercise XIb

(1) A merchant bought 50 pairs of shoes for $210.00 and retailed them at $5.60 per pair. What was his rate of profit?

(2) When the wholesale price of sugar is $8\frac{1}{3}$¢ per pound, how many pounds must be retailed for $1.00 to gain 20%?

(3) A stationer bought paper at $3 a ream and sold it at 25¢ a quire. What was his rate of profit?

(4) A merchant mixes tea worth 40¢, 60¢, 80¢ and \$1.00 per pound, the ratio of the weights of the different kinds in the blend being as 4: 3: 2: 1. He retails it at 80¢ per pound. Find his rate of profit.

(5) A merchant received a shipment of \$1,950 worth of goods which he marked to sell at a gain of 60% of the cost but reduced the marked price $\frac{1}{3}$ for a "bargain sale." What was his profit?

(6) A grain company bought 12,000 bushels of wheat at \$1.08 a bushel and 6,500 bushels of oats at 28¢ a bushel. In selling it gained $16\frac{2}{3}$% on the wheat and lost 20% on the oats. What per cent. did it gain or lose on the whole transaction?

(7) Fifty pounds of soap are purchased at 5¢ a pound. In a damp cellar it absorbs water equal to 10% of its weight. It is then retailed at 6¢ a pound. What is the rate of profit?

(8) I sell $\frac{3}{5}$ of a piece of land for what $\frac{3}{4}$ of it costs. Find the gain per cent.

(9) A dealer bought goods at 20% below the regular list price and sold them at 20% above the list price. What per cent. did he make?

(10) A retailer has found that the average cost of doing business is 20% of his gross sales for the year. His profits have averaged 25% of the gross sales. At what price must he sell stoves that cost him \$36 each to cover these items at the rate mentioned?

(11) If a dealer sells goods at a profit of 50% but fails to collect 10% of his sales, what per cent. of profit is he really making?

(12) I bought goods and sold $\frac{1}{4}$ of them at a loss of 20%. By what per cent. must I raise the selling price in order to gain 20% on the entire transaction?

(13) A merchant marks his goods at an advance of 25% on cost. He allows a customer a discount of 10% for cash and still makes a profit of $5.60 on the sale. What was the amount of the sale?

(14) By selling an article for $7.20 I gain $12\frac{1}{2}$%. What per cent. would I gain by selling it for $7.50?

(15) A boy buys 120 oranges at the rate of 4 for 30¢ and sells them at the rate of 3 for 25¢. What is his per cent. of gain or loss if 10% of the oranges are unsaleable?

(16) A dealer sells two wagons for $75 each. On one he gains 25% and on the other he loses 25%. What is his per cent. of gain or loss on the sale?

(17) A baker's outlay for flour is 65% of his gross receipts, and other expenses 25%. The price of flour rises 10%. What per cent. reduction must he make in the other expenses in order that he may make the same rate of profit?

(18) A dairyman's herd of cows produced 225 gallons of milk per week during four weeks of June. During four weeks of July he increased the amount of grain fed to his cows by $5 worth per week and obtained 15% more milk. If milk sells at 12 quarts for $1.00 ,what per cent. did he make on the new outlay by increasing the amount of grain fed?

(19) Suits are selling at $32.40 each, and at this price the average sale per day is 12. The sale price is reduced $12\frac{1}{2}$% whereupon the sales increase to 20 per day. If

the suits cost \$20.00 each, by what per cent. per day does the merchant increase his gain by the cut in price?

(20) A real estate agent bought a lot and sold it so as to gain 20%. With the proceeds he bought a second lot which he afterwards sold at a gain of $12\frac{1}{2}$%. If his total gain from the sale of the two lots is \$280, how much did he pay for each lot?

(21) A dealer adds $33\frac{1}{3}$% to the prime cost of a certain article to cover its share of the overhead expenses. He then sells the article at a profit of 15% on the total cost. If the prime cost is \$40, what is the selling price?

(22) A hardware merchant buys 50 stoves at \$36.40 each. He pays \$62.30 freight on the shipment, allows \$12.70 for other buying expenses, and estimates the overhead and selling expenses of each stove to be \$2.50. He sells the stoves at \$50.50 each. What is his per cent. of profit on the total cost?

(23) A manufacturer determines his selling price by adding 32% to the manufacturing cost. Find the cost of manufacturing an article which he sells for \$3.96.

(24) A dealer buys a car from the manufacturer for \$2,000. At what price must he sell the car so that he may be able to allow a purchaser a reduction of 20% for cash, and yet make a profit for himself of 40%?

(25) A man bought a house at 20% below its real value and sold it at 10% above its real value. If his profit was \$1440, how much did he pay for the house?

(26) The receipts of a company amounted in one year to \$354,000 and the expenses to \$251,000. In the next year receipts fell off by $3\frac{1}{2}$% and the expenses increased $\frac{1}{2}$%. Find the percentage decrease in the net profits.

(27) A candidate fails in his examination by 2% of the required minimum; he obtains 735 marks. If this required minimum is 50% of the total marks, what is the total?

(28) A man sold two lots at $1200 each. On one of the lots his gain was 20%. If on the whole he neither gained nor lost, at what per cent. loss was the second lot sold?

Exercise XIc

(1) A dealer buys goods catalogued at $300 subject to discounts of 20%, 10% and 5% and sells them for $250. Find his gain per cent.

(2) I can buy from one dealer a piano for $650 less $33\frac{1}{3}$% and 20%. Another dealer offers the same piano for $700 less $37\frac{1}{2}$%, 20% and 5%. Which is the better offer and by how much?

(3) Find the net amount of the following bill:—

150 yds. Tweed	@	$2.40
120 yds. Silk	@	3.50
240 yds. Gingham	@	.35
180 yds. Cotton	@	.25

Discounts of $33\frac{1}{3}$%, $12\frac{1}{2}$%, and 5%.

(4) Find the net amount of a bill of the following items:

$75 less 25% and 20%.
$240 less $16\frac{2}{3}$% and $12\frac{1}{2}$%.
$320 less $37\frac{1}{2}$% and 25%.

A cash discount of 5% is allowed on the whole bill.

(5) What single discount is equivalent to successive discounts of 20% and 10%?

(6) What single discount is equivalent to three successive discounts of 10%?

(7) A merchant buys goods listed at \$360 with successive discounts of 25% and 5%. He sells at 10% below the list price. What gain, and what gain per cent., does he make?

(8) A retail merchant bought goods invoiced at \$750 subject to discounts of 20% and $16\frac{2}{3}$%. He sold the goods at an advance of 40% on their actual cost. Find his gain if 10% of the sales could not be collected.

(9) Suits in the window of a gent's furnishing store are marked "\$46.50, selling for \$31.00." By what per cent. have the prices been cut?

(10) From the list price of a line of goods a purchaser is allowed a discount of 20%. A second discount of 10% is allowed for buying a large quantity and a third discount of 5% for cash. If he sells at 10% under the list price, find his rate of profit.

(11) A bookseller allows a discount of 5% to all customers for cash but allows teachers a further discount of 10% off all cash prices. A teacher's bill amounted to \$8.55. Find the total marked price of the goods bought.

(12) A merchant marks his goods at 40% above cost and in selling uses a pound weight $\frac{1}{8}$ oz. too light. If he sells at 10% off his marked price, what will be his rate of profit?

(13) A hardware merchant deducts 10% from the marked price of his goods and yet realizes a gain of 25%. Find the cost to the merchant of a lawn-mower that he sells for \$14.40. At what per cent. is the marked price in advance of the cost price?

(14) What must be the marked price of goods which cost \$12.00 in order that the merchant may allow a discount of 20% and still make a profit of 25%?

(15) Olives are selling at 40¢ a bottle or \$9.00 per case of two dozen bottles. What per cent. is saved by buying by the case?

(16) If an ore loses $37\frac{1}{2}$% of its weight in roasting and 52% of the remainder in smelting, how much ore will be required to yield 1500 tons of metal?

(17) A customer induces a merchant to give him a discount of 10% off the marked price for cash. The goods were marked at an advance of 15% on the cost. The merchant's profit on the sale is \$1.40. Find the cost price, the marked price, and the selling price.

(18) A merchant marks his cotton at an advance of 20% on cost and after selling one-half of it, finds that one third of the remainder is so damaged as to sell for only one half of its cost. What advance must be made on the marked price of what now remains so that on the whole there may be a gain of 15%.

(19) At what price must goods which cost \$202.50 be listed to give a gain of 20% after allowing 25%, 20% and 10% discounts?

(20) A manufacturing company lists a line of men's suits at \$63.00 each and allows discounts of 20% and 10%. At what price each must a retailer mark these suits in order that he may make a profit of 15% after allowing a customer a discount of 10%?

Exercise XId

(1) An agent buys a section of land for his client at \$35 per acre and charges $2\frac{1}{2}$% commission. How much

does the agent receive for his work and what is the total price the client pays for the section?

(2) An agent sold 3000 tons of hay at \$12.50 per ton. If he charges 5% commission find the sum remitted to the consignor.

(3) An agent bought a piece of land and charged \$180 as his commission, the rate being $2\frac{1}{4}\%$. How much did the agent pay for the land?

(4) The Acme Elevator Company arranges for the purchase of 12,000 bushels of wheat at 85¢ a bushel. If the rate of commission is 2%, what sum must be sent to the buyer to complete the purchase and pay the commission?

(5) A farm was bought for me on a commission of $2\frac{3}{4}\%$. If the farm cost me \$4,932, how much commission did I pay?

(6) I remitted \$2,575, including commission, to my agent to purchase goods for me on a commission of 3%. How much did he pay for the goods?

(7) An agent buys goods for \$2,745 and charges his customer \$2,806 for them. What is his rate of commission?

(8) A salesman receives a monthly salary of \$160 and, in addition, a commission of 5% on his sales. What must be the amount of his yearly sales to give him an income of \$3,600?

(9) My agent bought a carload of potatoes (750 bu.) for me at 80¢ per bushel on a commission of 2%. If freight, draying and other expenses amount to 15¢ a bushel, for how much per bushel must I sell them to make 20%?

(10) A commission merchant sold for a consignor 650 pounds of butter at 30¢ a pound. The charges were freight \$4.43 and commission $2\frac{1}{4}\%$. He invested the proceeds in coffee at 28¢ a pound, charging $1\frac{1}{2}\%$ for buying. How many pounds of coffee did he buy?

(11) If an agent remits \$625.38 as the net proceeds from the sale of goods after deducting \$18.72 for freight and 5% for commission, how much was the commission?

(12) An agent received a consignment of wheat which he sold at 80¢ a bushel, charging a commission of 2%. With the net proceeds, after reserving a commission of 2% for buying, he purchased lumber at \$23.00 per M. If the selling commission exceeded the buying commission by \$9.20, find the quantities of wheat sold and lumber bought.

(13) At what price must I mark an article which cost me \$27 so that I can allow 25% discount for cash, pay an agent 15% of the list price as commission for selling, allow 5% of sales for bad debts, and still make a profit of $33\frac{1}{3}\%$?

(14) I sent my agent \$3,451 to invest in tea at $56\frac{2}{3}$¢ per pound after deducting his commission of $1\frac{1}{2}\%$. How many pounds did he buy? At what price per pound must the tea be sold to make a profit of 20%, the freight charges amounting to \$39.50?

(15) I sent my agent \$2,728 to invest in apples at \$4.00 per barrel and to pay all expenses of the purchase. The expenses were as follows:—Commission 5%, freight 15¢ per barrel, draying 5¢ per barrel. Find the number of barrels bought.

(16) Having sold a section of land on 5% commission I receive instructions to invest the proceeds in city lots

after deducting my commission of 2% for buying. My total commission is $280. How much did I invest in city property?

(17) An agent charges the same rate of commission for selling and buying. He sells a carload of wheat for $2,030, and after deducting $60 for the two commissions, invests the proceeds in cattle. Find the rate of commission charged.

(18) An agent bought a tractor for me at discounts of 25% and 10% off the list price and charged me 5% for buying. The tractor cost me $945. What was the list price? What was the amount of commission.?

(19) An agent sold 6500 bushels of potatoes at 60¢ a bushel, charging 5% commission. The freight charges were 2¢ a bushel and other expenses amounted to $85. He invested $2,475 in silks, charging 5% for buying, and remitted the balance of the proceeds. What was the amount of the remittance?

(20) A commission merchant had shipped to him 1200 barrels of flour and 5000 bushels of wheat. He paid $12\frac{1}{2}$¢ a barrel for storage of the flour and 3¢ a bushel for storage of the wheat. He also paid $84.56 freight charges. He sold the flour at $6 a barrel on a commission of $1\frac{1}{2}$% and the wheat at 90¢ a bushel on a commission of 2¢ a bushel. What sum should he remit?

CHAPTER XII

TAXATION

A. Municipal Taxes

A municipality requires money to build roads and bridges, make various other improvements, support schools, give police and fire protection, pay administration expenses, etc. This money is collected as Taxes. Part of this may be raised in special ways such as sale of licenses, rent or sale of property owned by the municipality, or by various fees, but the bulk of it is collected from those who own property in the municipality, or living within it, are in receipt of income. All property within the municipality is valued or *assessed* by employees of the municipality. Except in case of property exempted from taxation for some reason, a certain fraction of the assessed value is collected from the owners for the purposes named. The rate of taxation is generally expressed as mills on the dollar. Since a dollar contains 1000 mills, a rate of 21 mills on the dollar would be 21/1000 of the assessed value. Improvements such as road construction, putting down pavements, sidewalks etc., may be paid for in whole or in part by an acreage tax or a tax per foot on property frontage. These taxes are paid by the property holders concerned in addition to the regular tax rate.

A tax notice specifies the mill rate levied for each purpose, the total rate and the amount of taxes this rate calls for.

Example:—What tax does a man pay whose property is assessed for $8,500, the tax rate being 36 mills?

36 mills on $\$8500 = \frac{36}{1000}$ of $\$8500 = \$306$.

B. Provincial Taxes

The taxes imposed by the provincial governments are different in different provinces. Some of the taxes imposed are: succession duties, amusement tax, corporation tax and taxes on automobiles.

Succession duties are taxes levied on property passing to individuals or institutions on the death of the owner of the property. These duties are imposed as a percentage of the value of the estate, providing its value is in excess of a specified sum. The percentage rate increases as the value of the estate increases and also as the kinship of heirs becomes more remote. It is higher in the case of property passing to a brother, sister, uncle, aunt, or descendent of these than when left to a grandparent, parent, child, husband or wife. If passing to more remote relatives or to those not relatives of the deceased, the rate is still higher and the minimum value of an exempted estate is lower.

An amusement tax is levied in some provinces on persons attending places of amusement to which an admission fee is charged. This tax increases with the price of admission but does not exceed a specified maximum.

A corporation tax is a tax on companies or corporations (other than municipal) doing business within the province. The amount of the tax varies with the nature and volume of the business carried on. Banks, electric lighting companies, insurance, gas, grain, express, land, loan, power, telephone, telegraph, and other companies are subject to this tax.

Automobile owners pay a special tax in the nature of a registration fee, which increases with the length of the wheel base of the car. A per gallon tax on gasoline sold in the province is also borne for the most part by users of motor vehicles.

C. Dominion Taxes—Direct

The Dominion Government levies taxes of various kinds, some direct and others indirect. Among the former are a stamp tax and an income tax.

Every receipt for an amount of $10 or over is required to have a 2 cent revenue stamp affixed, and every cheque for an amount greater than $5, must have affixed stamps to the value of 2 cents for every $50, or fraction thereof, of its face value. No cheque, however, requires stamps in excess of a maximum of $1.

The Dominion Income Tax is levied (1925) on all incomes in excess of a statutory exemption of $3000 in the case of companies, married persons, or unmarried persons having dependents. In the case of unmarried persons, widows, or widowers, without dependents the exemption is $1500. $1500

An income tax levy is made up of one or all of the following parts and is calculated on the official forms in the manner here indicated.

(1) *Normal Tax.* This is a tax of 4% on income in excess of the statutory exemption until the net income reaches $6000, and 8% on all income in excess of $6000.

From this is deducted the tax on income derived from Canadian enterprises on which income tax is levied direct, and on $500 for each dependent under 18 years of age. The allowance is 8% in each of these cases if the net income exceeds $6000 by an amount sufficient to cover the above

2% up to $3000
3% " " $3000 etc
20% up to $90 000 and over

dividends and the $500 for each child. If not covered by the income in excess of $6000, 4% is allowed on that portion not so covered, or on all if the income is less than $6000. Dividends from tax free Victory Bonds are not considered part of the net income for purposes of the income tax.

(2) *Surtax.*—This is levied on all net incomes over $5000 according to a graduated scale. 1% is levied on the amount of income in excess of $5000 until $6000 is reached, 2% is levied on the income in excess of $6000 until $8000 is reached, 3% on $8000 to $10,000, 4% on $10,000 to $12,000, etc., until $100,000 is reached, after which the rate of increase is less rapid.

(3) *Additional Tax.*—This is levied on incomes over $5000 and is equal to 5% of the sum of the normal tax and surtax.

Examples:

(1) What income tax is paid by a married man with an income of $4500, if he has two children under 18 years of age, and $150 of his income is from tax free Dominion War Loan Bonds?

Income	$4500
Dividends from tax free Bonds	150
Net Income	$4350
Statutory Exemption	2000
	$2350
Normal Tax, 4%	94
Exemption for children, $1000 @ 4%	40
Tax payable....................	$54

(2) Find the income tax payable by a single man without dependents, if his income is \$4500, of which \$250 is from Canadian Dividends.

Net Income	\$4500
Statutory Exemption	1000
	\$3500
Normal Tax, 4%	\$140
Canadian Dividends, \$250 @ 4%	10
Tax Payable	\$130

(3) A married man has a net income of \$7500. He has one child under 18 years of age. What is his income tax?

Net Income	\$7500
Statutory Exemption	2000
	\$5500
Normal Tax (4% on \$4000+8% on \$1500)	280
Exemption for child, \$500 @ 8%	40
Net Normal Tax	\$240
Surtax, (1% on \$1000+2% on \$1500)	40
Additional Tax, 5% on \$280	14
Total	\$294

(4) A married man with 4 children under 18 years of age has a net income of \$6500. What is his income tax?

Net Income	\$6500
Statutory Exemption	2000
	\$4500
Normal Tax (4% on \$4000+8% on \$500)	200
Exemption for children, \$500 @ 8%+\$1500 @ 4%	100
Net Normal Tax	\$100
Surtax 1% on \$1000+2% on \$500	20
Additional Tax (5% on \$120)	6
Total	\$126

D. Dominion Taxes—Indirect

Custom houses are placed at "ports of entry" to this country, and at these goods entering the country are examined and either admitted without any entry tax or subjected to certain taxes called import duties. The ports of entry are situated at convenient places for distribution all through the country. The goods are shipped to these places and are kept under seal until released by the proper government officials.

Duties levied on goods according to measure, weight or number, irrespective of value, are called *specific duties.* Duties levied on the cost of the goods in the country in which they are purchased, as shown by a

certified invoice are *ad valorem duties.* An ad valorem duty is quoted as a percentage of the invoice price. Goods may be subject to either or both of these duties.

Occasionally duties are levied on goods shipped out of the country, especially on raw materials. These duties are calculated in the same way as the import duties but are called *export duties.*

Some commodities such as tobacco and spirits are taxed when manufactured in the country, and sold for consumption within the country. This tax is called *excise duty,* and is paid when the goods are released from bonded warehouses.

A tax, known as the sales tax, is collected when all goods, except certain specified articles, are sold by the producer or manufacturer. It is 5% of the price of the goods. In the case of imported goods it is 5% of the "duty paid price," that is, of the invoice price of the goods increased by the duty, and is paid by the importer when he sells the goods. The sales tax is an excise duty.

Examples:

(1) Find the duty on 5200 lbs. of sugar worth 8 cents a pound, the specific duty being $\frac{1}{2}$ cent a pound and the ad valorem duty $17\frac{1}{2}\%$.

5200 lbs. sugar @ 8 cts	\$416.00	
Ad valorem duty $17\frac{1}{2}\%$		\$72.80
Specific duty, $(\frac{1}{2}\times 5200)$ cts.		26.00
Total duty		\$98.80

(2) As a consequence of a reduction in duty from 35% to 25% the importation of a commodity increases 50%. How is the revenue from the duty affected?

On goods invoiced at $1000

Duty at 35% would be $350

Increased importation $1500

Duty at 25% $375

On $350 there is an increase of $25

On $100 there is an increase of $\$\frac{2500}{350} = \$7\frac{1}{7}$
Increase in revenue $7\frac{1}{7}\%$.

Exercise XIIa

(1) In a town where the assessed valuation of property is $1,350,000, the revenue from taxation is $27,000. What is the rate of taxation expressed (1) in mills on the dollar, (2) in per cent.? What amount of taxes will a man have to pay who owns property assessed at $4500?

(2) My house is valued at $5600 and my lot at $1500. The house is assessed at 35% of its value, and the lot at 80% of its value. If the tax rate is 45.5 mills, find the amount of my taxes.

(3) A man owns property valued at $8500 which is assessed at 60% of its valuation. His tax amounts to $85. Find the rate of taxation.

(4) A man receives a salary of $3500. His net income, after paying an income tax on all over $2000, is $3440. Find the rate of taxation.

(5) \$1500 of a man's annual income is exempt from taxation. The tax rate is 15 mills on the dollar. What is his annual income if his net income after paying the tax is \$3174.50?

(6) A person who pays an income tax at the rate of 4% on all his income above \$2500 receives a tax bill for \$56.40. Find his income.

(7) The estimated expenditures of a rural school district for the year 1923 were: Teacher's salary \$1100.00, Secretary-Treasurer's salary \$75, caretaking \$50, fuel \$95, repairing buildings and fences \$30, debenture payment \$145. The district's revenue apart from taxes was: government grant \$180, rent from school building for community meetings \$45. If this district is 5 miles square, what was the average school tax on each quarter-section of land in the district?

(8) The details of the tax levy in a city for the year 1922 were as follows:

Municipal rate	7.3	mills
School rate	9.5	"
Hospital rate	1.4	"
Library rate	.35	"
Parks rate	.75	"
Debenture rate and other interest charges	12.7	"

In the case of taxes remaining unpaid after June 30th, 1922, a penalty of 5% of the amount of the taxes was added for the month of July and afterwards a sum equal to 1% per month, or fraction of a month, until such taxes were paid.

The owner of property assessed at \$3520 paid his taxes on Oct. 20th, 1922. How much did he have to pay?

(9) A town requires \$15,200 to meet expenditures for the year. 5% of the taxes is allowed for collection. What rate must be struck if the assessed value of the taxable property is \$1,200,000?

(10) A school district borrowed by debentures \$3000 to build and equip a school, this sum to be paid back in 10 equal annual instalments together with interest at 6% per annum. If the assessed valuation of the property in the district is \$280,000, find the rate of taxation in mills necessary to meet the payment of the fourth debenture.

(11) A merchant imported 20,000 pencils, net weight 240 pounds, paying for them \$3.75 per hundred. Freight and other expenses amounted to \$12.50. The duty on pencils is \$2.50 per pound and 25% ad valorem. What rate of profit would he make by retailing the pencils at the rate of 8 for \$1.00?

(12) A Vancouver merchant bought 600 yards of silk invoiced in New York at \$3.60 a yard, on which he paid an ad valorem duty of $12\frac{1}{2}\%$ and a specific duty of 5 cents a yard. At what price per yard must he mark the silk that he may allow 10% discount for cash and still make a profit of $33\frac{1}{3}\%$?

(13) When the rate of taxation is $37\frac{1}{2}$ mills on the dollar find the assessed value of property on which the tax is \$150.

(14) If goods invoiced at \$1670 in New York cost \$2100 when laid down in Edmonton, the insurance, freight and cartage amounting to \$130, what was the rate of duty?

(15) A merchant imported 1500 yards of dress goods invoiced at $3.20 per yard on which he paid an ad valorem duty of $12\frac{1}{2}\%$. When unpacking the goods he found that 20% of them had been so damaged that he had to sell them at $33\frac{1}{3}\%$ below the invoice price. At what price per yard must he sell the remainder to make a clear gain of $540?

(16) The duty on imported vacuum cleaners is $39 per dozen and $16\frac{2}{3}\%$ ad valorem. The whole duty paid on a shipment of cleaners was $810, the ad valorem duty being $342 more than the specific duty. Find the invoice price per cleaner.

(17) R. Strome & Co. imported from Havana 75 barrels of molasses at $10 per barrel and 800 boxes of cigars (each box weighing 1 lb. and containing 100 cigars) at $8 per box. 6% was allowed for leakage on the molasses. Find the total duty at $33\frac{1}{3}\%$ ad valorem on the molasses and $1.50 per pound and 25% ad valorem on the cigars.

(18) The rate of taxation is 9 mills on the dollar. What is the value per acre of a half-section of land on which the tax is $163.20, if it is assessed for $\frac{5}{6}$ of its value?

(19) A rural school district having an assessed property valuation of $200,000 operates a school for 7 months at a cost of $840, and could operate it 2 months more at an additional cost of $180. What will be the increase in the taxes of a man owning property assessed at $2600, if a special tax is voted for maintaining the school 2 additional months?

(20) In 1923 my taxes were $144.00 on an assessment of $4800. In 1924 the assessment was 25% higher and

the taxes $12\frac{1}{2}\%$ higher. By what per cent. had the rate changed?

(21) A school board decides to build a school-house and lets the contract for \$9,120. The money is collected by a tax on the town property assessed at \$593,750. Find the rate of taxation to cover both the cost of the school and the collector's commission of 4%.

(22) A fifty foot city lot is assessed at \$1800, and the house on it, worth \$10,000, is assessed at 75% of its value. The tax rate is 48 mills, and the local improvement taxes are 3 cents a foot for boulevard, 8 cents a foot for sidewalk and 38 cents a foot for pavement. What tax does the owner pay?

Exercise XIIb

(1) A man, unmarried and without dependents, has a salary of \$150 per month. What is the amount of his annual income tax?

(2) James Kelly, unmarried, but supporting two dependents, each over 18 years of age, has a salary of \$250 per month. Find his annual income tax.

(3) A married man with two children under 18 years of age holds a position which pays \$10,000 per year. Find the amount of his yearly income tax.

(4) The salary of an unmarried man, without dependents, is \$5800, of which \$400 is from Canadian securities. Find the annual tax on this income.

(5) A married man with 3 children under 18 years of age pays an income tax of \$20. On what income was the assessment made?

(6) The following is a merchant's yearly financial statement of profit and loss for the year 1923 as required by the Income Tax Department:

Merchandise sold during the year	$60,000
Merchandise on hand at beginning of year at cost price	10,000
Cost of merchandise bought during year for sale	50,000
Merchandise on hand at end of year at cost price	11,000
Salaries and wages	1,300
Rents	600
Interest paid on borrowed money used in business	400
Fire Insurance premiums	600
Depreciation on Store Fixtures	400
Uncollectable accounts written off during year	600
Light, fuel and general operating expenses .	1,200

Find the amount of income tax the merchant had to pay in 1924, if he is a married man with no children under 18 years of age.

(7) A farmer's gross income for 1923 is made up of the following items:

Wheat sold, 3000 bu. at	$1.20	
Oats sold, 1500 bu. at	0.46	
Barley sold, 1200 bu. at	0.80	
Hay sold, 30 tons at	$20.00	
Potatoes, sold, 100 bu. at	0.75	5975
Hogs sold		$500.00
Eggs, butter and cream		450.00
Value of products of farm consumed on farm		300.00
Hail Insurance received		800.00
Salary received from Secretaryship of Zebma S.D.		150.00
		8125

Deductions from gross income:

Seed purchased	$600.00
Feed purchased for stock	400.00
Binder twine purchased	200.00
Repairs to machinery	80.00
Repairs to fences	60.00
Insurance premiums paid on livestock, machinery and buildings	80.00
Hail Insurance premium	260.00
Taxes on farm	140.00
Threshing	960.00

Depreciation: Frame buildings, cost $3000; rate 5 per cent.
Machinery, cost $2000; rate 10 per cent.
Work horses, cost $1000; rate 10 per cent.

On the basis of the above Tax Return, find the amount of income tax this farmer pays in 1924, if he is married and has 3 children under 18 years of age.

(8) A married man with 4 children under 18 years of age has a salary of $7500. Find the amount of his income tax.

Exercise XIIc

(1) An estate in Alberta valued at $60,000 is willed to three sons residing in the province, each son receiving an equal share. If the rate of Succession Duties Tax is $2\frac{1}{2}\%$, find what each son receives.

(2) An auto dealer buys a car invoiced at $1250, subject to a sales tax of 5%. He pays freight $32.50 and sells it to a customer at an advance of 30% on the total outlay. The buyer pays by cheque with proper stamps affixed. What is the cost to the buyer?

(3) A Loan Company doing business in Alberta is capitalized at $75,000 and its gross revenue for a year is $36,000. The Provincial Corporation Tax is $50 if the capital of the company is between $50,000 and $100,000, and $\frac{1}{2}$ of 1% of the gross revenue. Find the amount of Corporation Tax this company pays.

(4) An estate valued at $84,000 is left to the widow and three daughters resident in the province. The widow is to receive $30,000 net, and the balance after deducting all succession duties from it is to be divided equally among the three daughters. If the Succession Duties Tax rate is $3\frac{1}{2}$% find the amount of each daughter's share.

(5) A wholesale house imports dry goods invoiced at $480, pays an ad valorem duty of 20% and pays $16 freight on the shipment. The goods are sold to a retailer at an advance of 20% on the total cost. The retailer pays the sales tax of 5% on the duty paid price, and makes 30% on his total cost. What does the consumer pay? How much of this goes to the government?

(6) A firm in Canada imports the following goods from England:

	Invoice Price in Dollars	*Discounts*	*Duty*
Woollen Dress Goods	$4000	20% and 10%	15%
Knitted Goods	$2000	15% and 5%	20%
Silk and Velvet Fabrics	$1800	25% and 10%	$17\frac{1}{2}$%
Boots and Shoes	$2400	20% and 5%	$17\frac{1}{2}$%

The Sales Tax is 5% of the "duty paid value" of the goods. Find the amount of Sales Tax on this bill of goods.

(7) A resident of Alberta dies and leaves an estate valued at $30,000. Of this estate $25,000 net is bequeathed to the widow, and the balance after all succession duties have been deducted passes to a distant relative. If the tax rate on the widow's portion of the estate is 1% and on the balance 4%, how much does the distant relative receive?

(8) A Motor Sales Company in Moose Jaw imports a car-load of six automobiles from Detroit, U.S.A. The invoice price of each automobile is $1000 less 20% and 10% discounts. The Excise Tax in the United States is 5% of the net sale price and the duty is 35% of the invoice price plus the American Excise Tax. The Sales Tax in Canada is 5% of the "duty paid value" of goods imported.

(1) Find the amount of Sales Tax paid on this car-load of automobiles.

(2) If the freight on this car-load amounted to $570, find the net cost of each automobile laid down in Moose Jaw.

CHAPTER XIII

INSURANCE

A. Fire and Marine Insurance

A fire insurance company agrees to pay a definite sum in case property is destroyed by fire, or an amount not exceeding this sum, to cover the loss in case of partial destruction by fire. For taking this risk the company receives a certain percentage or fraction of the sum named. The agreement given by the company to the insurer is called the *policy*. The amount the company agrees to assume liability for is the *face of the policy*. The sum paid the company by the owner or insurer is the *premium*. Insurance policies are generally issued to cover periods of 1 or 3 years, but may be issued for any specified period. The premium is a certain rate on the face of the policy, perhaps quoted as a percentage, or often as the number of cents paid for each $100 of the face of the policy.

Example.

A insured a business block worth $85,000, for $50,000, for 1 year paying a premium of $\frac{3}{4}\%$. Find the amount of the premium and the loss to the company and to the owner in case the block is burned during the year.

Premium is $\frac{3}{400}$ of $50,000	$375	
In case of total loss owner receives	$50,000	
The company loses	$50,000	
Less premium	$375	
Net loss		$49,625

The owner loses the block	\$85,000	
The premium	\$375	
	\$85,375	
Less Insurance	\$50,000	
Net loss		\$35,375
Check		\$85,000

The rate in this case might have been quoted as 75¢ on \$100.

If two or more companies carry risks on the same property, the loss in case of fire is divided between the companies so that all pay the same proportion of the risk carried.

A company taking a large risk frequently re-insures part of it in one or more other companies. Then in case of loss the first company pays the owner the face of the policy given, but is partly reimbursed by the companies carrying the re-insurance. The first company's net loss is the amount by which the original premium added to the new insurance falls short of the amount of the original policy together with premiums for re-insurance. The losses to the other companies may be found just as though the risks were taken directly from the owner.

B. Life Insurance

The forms of life insurance that may be carried are many and various. The majority of the policies, however, belong to one of three types:

(1) Whole Life Policies, premium payments for life. In this case the assured pays premiums as long as he lives and at death the amount of the policy, with or without "profits," is paid to his heirs.

(2) Whole Life Policies, limited payments. The assured pays premiums until the expiration of a definite term and has no more pay, but the insurance is not payable until death. A policy on which the payments are limited to 20 years is commonly designated a "20 pay life" policy.

(3) Endowment Policies. In this case the premium payments are limited to a certain number of years and the policy becomes payable at the end of this period (or a longer period) if the assured does not die in the meantime, in which case the insurance is payable.

To ensure financial solvency under varying conditions and thus to protect the policy contracts of the insured, life insurance companies charge premiums somewhat greater than are necessary to provide for the payment of death claims, expenses of the head office and agencies, and dividends on capital stock. The part of the premiums in excess of what is required for the purposes specified varies with the death rate, interest the company makes on its funds, etc. This excess is distributed from time to time as "profits" to policy holders. Policies sharing in these profits are called participating policies. A lower premium rate can be secured if the policy holder foregoes the profits. His policy in this case is non-participating.

"Profits" may be taken in cash, may be used to reduce premiums, or to purchase additional insurance, at the option of the policy holder.

Sample Tables

I *Whole Life Policy* *$1000*

Age	*Payments for life*	*20 Payments*	*10 Payments*	*5 Payments*	*1 Payment*
18	18.50	30.85	49.90	88.75	401.55
19	18.70	31.30	50.65	90.05	407.10
20	19.20	31.80	51.30	91.35	408.30
21	21.50	32.25	52.50	92.70	410.50
25	22.45	33.45	55.50	98.45	440.90
30	26.65	37.70	60.30	106.80	477.50
35	30.60	41.50	66.15	116.75	515.50
40	36.05	46.45	73.05	127.60	561.80
45	43.20	52.65	81.45	141.90	603.50
50	52.95	61.10	91.70	157.40	671.80
55	66.40	72.70	104.05	175.30	740.25
60	84.90	88.95	119.75	188.85	801.70

II *Endowment Policy* *$1000*

Age	*Due in 20 yrs.*	*Due in 25 yrs.*	*Due in 30 yrs.*	*Due in 40 yrs.*
18	52.50	43.25	36.60	28.90
20	53.80	43.50	36.85	29.25
21	53.95	43.65	36.95	29.45
25	54.35	44.10	37.60	30.35
30	55.20	45.10	38.75	32.05
35	56.45	46.55	40.55	34.65

40	58.35	48.85	43.35	38.65
45	61.45	52.65	47.90	
50	66.50	58.80	55.10	
55	74.75	68.45		
60	87.85			

Example:

A man 35 years old insures his life. If he takes a policy for $1000, premium payments to run for life, he pays $30.60 a year, using the foregoing tables for illustration purposes. If his premiums are to cease at the end of 20 years he pays $41.50 a year, if at the end of 10 years he pays $66.15 a year, etc. If he wishes to receive his insurance when he is 60 years old he takes a 25 year endowment policy and pays $46.55 a year.

C. Other kinds of Insurance

Automobiles may be insured against fire, theft or accident. Plate glass windows may be insured against breakage. The Post Office Department will insure parcels sent by mail. Insurance may be carried to cover loss due to sickness, or contingencies of various kinds such as war or other disaster. Insurance may be carried to cover loss owing to accident to the person of the assured. This secures a certain percentage of the policy as a weekly indemnity during temporary disability due to accident, a specified sum in case of permanent partial disability or the whole sum in case of permanent total disability, or death due to accident. Policies of this kind are called accident policies, but they sometimes include disability due to certain specified contagious diseases.

Another common form of insurance in Western Canada is hail insurance. A farmer may insure his crop

against damage by hail, taking out a policy covering any specified acres of crop, for any amount per acre. In case of total loss from hail he receives the full insurance. In case the estimated damage is 50% of the crop he receives 50% of the insurance carried. If the damage is 80% he receives 80% of the insurance, etc. For this insurance he may pay 6% of the policy, or perhaps as high as 12%, if the crop is in an area frequently visited by hail. These general conditions and rates vary with different insurance companies.

Example:

Find the premium paid and the insurance received on a crop of 120 acres of wheat, insured at $25 per acre at 7% premium, and damaged by hail to the amount of 40%.

Insurance carried $\$25 \times 120$	$3000
Premium $\frac{7}{100}$ of $3000	$210
Insurance payable $\frac{40}{100}$ of $3000	$1200

Exercise XIII

(1) A dwelling valued at $3600 was insured for $\frac{2}{3}$ of its value at $\frac{3}{4}$%, and the contents, valued at $3000, for $\frac{3}{4}$ of their value at $\frac{4}{5}$%. A fire causes a total loss of the building and an 80% loss on the contents. What will be the amount of the owner's claim against the insurance company? Find the amount of the premiums paid.

(2) A merchant insures his building worth $32,000 for 80% of its value at $1.50. Find the premium.

(3) Find the cost of insuring a stock of goods for $15,000 for 8 months at $1.10 per $100 per annum.

(4) A building valued at $12,800 was insured for a period of three years for $\frac{3}{4}$ of its value, the rate being $1\frac{1}{2}\%$. Shortly after the insurance had been effected the building was completely destroyed by fire. Find the insurance company's loss; the owner's loss.

(5) A company insures a building for $16,000, the rate being $1.00 (per $100), and reinsures in a second company $\frac{2}{5}$ of the risk at the rate of $1.10. If the building is completely destroyed by fire, what is the amount of each company's loss?

(6) An importer insures 1500 barrels of flour for 80% of its cost at $2\frac{1}{2}\%$. The amount of the premium was $120. What was the purchase price of the flour per barrel?

(7) A merchant insures his stock for $\frac{7}{8}$ of its value at $1.50. If the premium paid is $420, find the value of the stock.

(8) A merchant has his stock insured in Company A for $6000, in Company B for $5000 and in Company C for $9000. A fire damaged the stock to the extent of $10,000. Find the amount of his claim against each company.

(9) Property valued at $15,000 is insured for $\frac{4}{5}$ of its value at 75¢ per $100 per year. If the policy is cancelled at the end of nine months by the insurer, how much of the premium should be returned?

(10) A ship's cargo valued at $80,000 was insured for $64,000 in one company at $\frac{7}{8}\%$ and for $16,000 in another company at $1\frac{1}{8}\%$. What per cent. premium was paid on the whole risk?

(11) A shipment of flour was insured at $\frac{3}{4}\%$ to cover two-thirds the value of the flour and the premium paid. If the premium paid was $135, what was the value of the flour?

(12) A building is insured for $500 more than $\frac{4}{5}$ of its cost at $1\frac{1}{2}\%$. The premium paid is $247.50. Find the cost of the building.

(13) A farmer insured his wheat crop of 320 acres against damage by hail in each of three different Hail Insurance Companies at $10 per acre. The rate of premium charged by the first company was 7%, the second company 10%, and the third company 12%. Find the total amount of the premium paid by the farmer for this protection. During the month of July his crop was partially destroyed by hail, the companies' adjusters estimating the damage at 75% over the entire area. Find the total amount of insurance received by the farmer.

If the crop averaged 15 bushels to the acre when threshed and the wheat is sold at $1.05 a bushel, find the farmer's net return per acre.

(14) At the age of 25 years the annual premium on a twenty-pay-life policy is $35 for $1000. Every five years the dividends decrease the premium by 10% of the original premium. What would it cost to carry $6000 of such insurance for 15 years?

(15) On January 2, 1920, R. Brown insured his house worth $4800 in the London and Lancashire Fire Insurance Company for 80% of its value at a yearly premium of $1\frac{1}{2}\%$. During the third year the house was destroyed by fire. Find the owner's net loss and also the net loss of the insurance company.

(16) I insured my motor car valued at \$1600 for 90% of its value against fire, theft, and collision. The fire rate is 1%, the theft rate $\frac{3}{4}$%, and the collision rate $2\frac{1}{2}$%. Find the amount of premium paid for this protection.

(17) A company insures an apartment house for \$60,000 at $1\frac{1}{2}$%. It reinsures in another company 30% of the risk at $1\frac{1}{4}$%, and in a third company 25% of the risk at 1%. What rate of insurance did the first company obtain on the amount of risk retained?

(18) An accident insurance company insures a group of 20 men for \$2000 each, on a premium of \$7.50 per \$1000 per annum. In the next 10 years, one of the men is killed in an accident after the payment of 10 premiums, another receives an indemnity of \$10 a week for 33 weeks, another is allowed a cash indemnity of \$250, and a fourth dies from natural causes after the payment of 4 premiums. What percentage of the total premiums is paid back in indemnities during this period?

(19) A farmer has 80 acres of wheat which he insures against hail for \$25 an acre at 8% premium. 40 acres of it is damaged to the extent of 60% by hail. He threshes 14 bushels per acre from the damaged portion, and 35 bushels per acre from the rest. If it costs him 15 cents a bushel to thresh and market the wheat, which 40 acres give the better return and by how much, if the wheat sells at \$1.20 per bushel?

(20) A married man 30 years of age takes out a 10-pay-life policy for \$5000, paying an annual premium of \$60 a year per \$1000. After paying 5 premiums he dies and his family receive the face of the policy and \$380 in profits. How much better off are they than would have

been the case had the premiums been invested and given a return equivalent to 8% per annum?

(21) At 20 years of age, a man takes out a whole life policy, unlimited payments; at 25, a 25 year endowment policy; at 30, a 20-pay-life policy, and at 35, a 10-pay-life policy, each for $1000. At 40 years of age how much has he paid in premiums (see tables)? If profits have paid for additions to the policies of 40%, 20%, 50% and 35%, respectively, how much insurance does he carry at 40?

(22) What weekly saving would pay the premium on a 40 year endowment policy for $4000, taken at age 21 (see tables)?

(23) A man takes out an accident policy for $2000, paying a premium of $15 a year. After paying 4 premiums he is injured in an accident and receives from the company $10 a week for 62 weeks. How many dollars indemnity does he receive for every dollar paid the company?

CHAPTER XIV

SIMPLE INTEREST—BANKING

A. Simple Interest

Interest is money paid for the use of money. The sum on which the interest is paid is called the *principal*, and the sum made up of the principal and interest added together is the *amount*. The problem of chief moment in this is the finding of the interest on a loan for a definite period, and the amount at the end of the period.

Examples

(1) Find the interest on \$1240 for 63 days at 7% per annum.

$$\text{Interest} = \$1240.00 \times \tfrac{7}{100} \times \tfrac{63}{365} = \$14.98.$$

In finding the interest for a number of days that is not a simple fraction of a year it is not necessary to divide by a larger number than 73. If the number of dollars is even the sum may be expressed in cents and the zeros will strike off with the 100 under the rate per cent. If there be not a 5 to divide into 365, divide by the 5 first and then divide quotient by 73.

(2) Find the interest on \$115 from 11th Jan., 1925, to 13th April, 1925, also the amount at the latter date, allowing interest at the rate of 6% per annum.

To find the number of days:

Jan. 31 − 11 =		20 days.
Feb.		28 days
March		31 days
April		13 days
	Total	92 days

There is no interest due 11th January so only the remaining days in January are counted. Each day in April marks the addition of another day's interest so the 13 days are counted.

Principal	\$115.00
Interest $\$115.00 \times \frac{6}{100} \times \frac{92}{365} =$	1.74
Amount	\$116.74

If the time involved be more than a year, compute the interest for the number of full years first, and then add the interest for the extra days (occasionally months are used) to find the total interest.

Simple interest tables are constructed to give the the interest on a sum of money for any number of days (see page 294 and following pages).

Example:

Find the interest on \$7642 for 61 days at 8% per annum, using the tables.

Interest on \$7600 for 61 days	= \$101.611
Interest on 42 for 61 days	= .562
Interest on 7642 for 61 days	= \$102.173

The interest on \$42 = Interest on \$4200 divided by 100.

Note: If the principal be \$P, the rate r% per annum, the time t years and the interest \$I; $I = \frac{Prt.}{100}$ If any three of the quantities P, r, t, I, be known, the fourth can be found by this formula. In a problem in which the principal, the rate or the time, only, has to be found, the formula gives a direct and easy solution.

B. Banking

Banks receive money to be kept on deposit until required again by the depositor. They lend money to those giving satisfactory security, and they facilitate the sending of money from place to place.

Money deposited in a bank in "current account" bears no interest, but those having money deposited in such an account may pay bills by cheques and at the end of each month the cancelled cheques are returned to the depositor and serve as proof of payment. Money deposited in the "savings account" bears interest which is added half-yearly to the amount on deposit. Cheques on savings account may be honoured by the bank, but the cancelled cheques are not returnable.

The Bank of Commerce

Halifax, June 3, 1924.

Credit................*H. J. Willis*

One Hundred and Eight $\frac{35}{100}$ Dollars

7×1	*7*	
×2		
4×5	*20*	
8×10	*80*	
×20		
×50		
×100		
Silver	*1*	*35*
Cheques		
Total	*108*	*35*

Deposit Slip

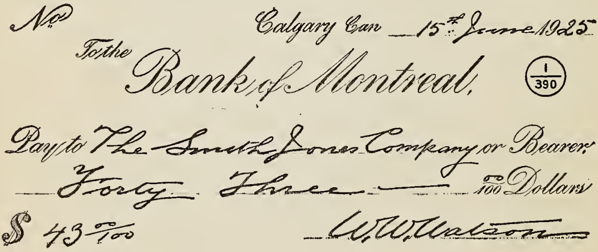
A "deposit slip" is made out when depositing money in the bank, specifying the number of bills of each denomination, silver, cheques, notes, or other cash equivalent deposited.

If the deposit is composed of eight \$10 bills, four \$5 bills, seven \$1 bills, and \$1.35 in silver, the deposit slip would be similar to the one shown.

Money may be withdrawn from a bank on filling out and presenting a cheque or a receipt for the proper amount.

When a payment is to be made by means of a personal cheque on current account the cheque may first be "certified" at the bank branch where the sender has money on deposit and the amount of the cheque is deducted from the cash balance on deposit. By "certifying" a cheque the bank guarantees its payment. If the cheque has not been certified the deduction is made when it is presented at the bank for payment. If the cheque is presented at some other bank it is returned by the other bank to the bank on which it is drawn, and the deduction is then made in the sender's account.

Banks often oblige their customers by cashing, or receiving on deposit, various money orders, or cheques on other banks, and themselves presenting them for payment at the institution on which they are drawn.


No. Calgary Can 15th June 1925
To the Bank of Montreal 1/390
Pay to The Smith Jones Company or Bearer
Forty Three 00/100 Dollars
$ 43 00/100 W. W. Watson

Cheque

In a centre where a large number of banks are located each bank cashes or receives from depositors, cheques on

the various other banks. Each bank then has a daily settlement to make with each of the other banks. To facilitate this settlement there is in such a centre an institution called a *clearing house* to which are brought daily all claims arising from the business of the previous day, of each bank against the other banks. A bank whose total claims exceed the claims against it receives the difference from the clearing house. A bank against which the claims are greater than the counter claims pays to the clearing house the difference. When these payments have been made, the amount received by the clearing house will be exactly equal to the amount the clearing house is called upon to pay and in this simple manner all claims are settled.

C. Bank Discount

A *promissory note* is an unconditional written promise to pay, for value received, on demand, or at some date in the future, a sum of money with or without interest. The sum named in the note is called the *face of the note.* If the note bears no interest the face of the note is collected at *maturity,* that is, when the date arrives on which payment has been promised. If it bears interest, the amount, that is, the face of the note, together with interest from the date of the note until the day of maturity, is collected.

Banks (and sometimes individuals) often buy notes before they are due. They pay the amount of the note at maturity, less simple interest on this amount, for the number of days between the date on which they buy it and the date on which it falls due. The deduction from the amount of the note is *bank discount,* and the sum paid for the note is called the *proceeds*

Ottawa, July 11th, 1923

Sixty days after date I promise to pay Thomas Wilson —or order, One Hundred and Fifty—$\frac{00}{100}$ Dollars at the Bank of Montreal here, with interest at the rate of seven per cent per annum.

Value received

150\frac{00}{100}$. Thomas Thompson.

The face of this note is $150.00.

The date of this note is 11th July, 1923.

The number of days until maturity is 63 (including days of grace).

The interest rate is 7%.

Date of maturity is 11th July+63 days, or 12th Sept., 1923.

The amount of the note is $150+interest on $150 for 63 days.

Face	$150.00
Interest $=\$150.00\times\frac{63}{365}\times\frac{7}{100}$	1.81
Amount	$151.81

If this note is sold to a bank, that is, discounted, on 2nd August, the bank pays $151.81, less interest on $151.81 from 2nd August to 12th September, 41 days. Suppose the rate of discount is 8%.

Amount of note	$151.81
Discount $=\$151.81\times\frac{41}{365}\times\frac{8}{100}$	1.36
Proceeds	$150.45

The bank pays $150.45 for the note.

If a note bears no interest the discount is calculated on the face of the note.

In the above problem the bank received at date of maturity \$151.81 or \$1.36 more than they paid for the note. This is for the use of \$150.45 for the 41 days. It would be slightly more than 8% on \$150.45, the sum the bank advances.

On \$150.45 for 41 days, bank receives \$1.36 interest.

On \$100.00 for 365 days, interest would be \$8.05.

Rate is 8.05% per annum on bank's investment.

Note: When the problem of finding the face value of a note has to be solved, given the discount or proceeds, labor may be saved by taking a note for \$730, or some other multiple of 365, calculating the discount or proceeds for this, and finding the true face of the note by proportion.

Exercise XIVa

(1) Find the simple interest in the following cases, the answer to be correct to the nearest cent.

	Principal	*Rate*	*Time*
(a)	\$1750	6%	$2\frac{1}{2}$ years.
(b)	\$ 960	8%	2 yr. 4 mo.
(c)	\$ 250	8%	3 yr. 9 mo.
(d)	\$4280	7%	5 yr. 3 mo.
(e)	\$3675	4%	4 yr. 2 mo.
(f)	\$ 735.50	$5\frac{1}{2}$%	146 da.
(g)	\$1428.60	$9\frac{1}{2}$%	292 da.
(h)	\$5680.25	8%	63 da.

(2) Find the interest on:

(a) \$630 from Jan. 7, 1923, to July 13, 1923 at 4%.
(b) \$7250 from Feb. 4, 1922, to Mar. 18, 1922, at 8%.

(c) \$1720 from Apr. 23, 1924, to Sept. 17, 1924, at $6\frac{1}{2}\%$.

(d) \$4320 from July 5, 1923, to Feb. 13, 1924, at 6%.

(e) \$3760.20 from Aug. 10, 1922, to Oct. 15, 1923, at $9\frac{1}{2}\%$.

(f) \$863.50 from May 23, 1921, to July 18, 1923, at 8%.

(3) A man who has been paying \$45 per month rent for a house, borrows \$6200 at 8% and buys a house. How much less is the interest paid than the rent?

(4) What principal will yield \$5.34 interest in 43 days at 5%?

(5) In what time will \$520 amount to \$532.48 at 6%?

(6) In 3 years \$644 amounted to \$788.90; find the simple interest rate.

(7) A merchant bought goods amounting to \$14,320; terms 3 mo., 5% cash. If he accepted the cash terms how much did he save, money being worth 6%?

(8) A man bought a house and lot for \$6000. He paid \$1200 cash and gave three notes of equal amounts for the balance due in 4, 8 and 12 months, with interest at $8\frac{1}{2}\%$. When he meets the last payment how much has the house cost him?

(9) A woman has \$20,000 invested in 6 houses which she rents at an average of \$32 a month each. Her yearly loss from vacant houses is \$300. Her annual income from rents is what per cent of the capital invested?

(10) A savings bank pays 3% interest on its deposits. It loans on bonds and mortgages at $6\frac{1}{2}\%$. If its deposits

amount on an average during the year to $15,375,000, and its loans amount to $13,850,000, what is its gross gain in interest per annum?

(11) On July 18, 1924, a man borrowed money at 8% interest and bought 4000 bushels of wheat at 85¢ a bushel. On Sept. 7 of the same year he sold the wheat at 98¢ a bushel and returned with interest the money he borrowed. What was his profit?

(12) A man borrows $2750 and agrees to pay $670 at the end of each year to meet the interest due at 8% per annum, and to reduce the principal. How much of the original debt remains after three payments have been made?

(13) The simple interest on a certain sum for $4\frac{1}{2}$ years at $3\frac{1}{2}\%$ exceeds that on the same sum for 3 years at 5% by $15. What is the sum?

(14) Two-thirds of my capital is invested at 3% and the balance at $4\frac{1}{2}\%$. My income is $420. What is my capital?

(15) The interest on a sum of money in $2\frac{1}{2}$ years is $\frac{5}{24}$ of the sum. Find the rate per cent.

(16) In what time will $250 double itself at 8% simple interest?

(17) The rent of a house at $40 per month pays the taxes at $1\frac{1}{2}\%$ of the value of the house, and $6\frac{1}{4}\%$ on the investment. How much money have I invested in the house?

Exercise XIVb

(1) Find the discount and cash proceeds of the following notes:

(a) A note drawn Feb. 3, 1923, at 90 days for \$540 and discounted March 19, 1923, at the rate of 6% per annum.

(b) A note made Dec. 13, 1922, at 3 months (days of grace included) for \$750 and discounted Jan. 5, 1923, at the rate of 8% per annum.

(c) A note made Jan. 15, 1924, at 60 days for \$1460 and discounted Jan. 19, 1924, at $7\frac{1}{2}$% per annum.

(2) Find the date of maturity, the term of discount, the proceeds, and the discount of the following notes:

(a) \$1500 Regina, June 1, 1921.

Six months after date I promise to pay G. Day, the sum of One Thousand Five Hundred Dollars, at the Bank of Montreal here. Value received.

James Kerr.

Discounted Aug. 17th, at 6%.

(b) \$4000 Calgary, Nov. 28, 1922.

Three months after date I promise to pay John Ross, or order, the sum of Four Thousand Dollars, at the Imperial Bank here. Value received.

William Hill.

Discounted Dec. 2, 1922, at $7\frac{1}{2}$%.

(c) \$2920 Vancouver, Feb. 8, 1923.

Sixty days after date I promise to pay to the order of Robert Johnston, the sum of Two Thousand Nine Hundred and Twenty Dollars, at the Royal Bank here. Value received.

Henry Scott.

Discounted immediately at 8%.

(d) $7300 Calgary, Mar. 10, 1924.

Ninety days after date I promise to pay to the order of James Anderson the sum of Seven Thousand Three Hundred Dollars, at the Bank of Nova Scotia here, with interest at 6% per annum. Value received.

Robert Kelly.

Discounted Apr. 15, 1924, at 8%.

(e) $4500 Halifax, Sept. 25, 1921.

Sixty days after date I promise to pay John Brown the sum of Four Thousand Five Hundred Dollars, at the Bank of Toronto here, with interest at 7% per annum. Value received.

Samuel Harris.

Discounted immediately at 8%.

(3) On Apr. 15, 1923, a man, by giving his note due in 3 months, borrows from the bank $500 cash. For what sum was the note made, if the bank's rate of discount is 8%?

(4) Find the face value of a note made August 16, 1920, at 60 days, and discounted the same day at 6%, if the proceeds amount to $469.20.

(5) A note drawn June 17, 1922, at 60 days, for $640, bearing interest at 6%, was discounted by the bank on July 3, 1922, at 8%. What rate of interest did the bank make on the money advanced? Had the face of the note been $730, what would the rate have been?

(6) A note of $1460, discounted 60 days before it was legally due, yielded $1442. What was the rate of discount?

(7) The discount on a note, made Feb. 27, 1923, at 3 months, for \$1080, with interest at 5% per annum and discounted Mar. 18, 1923, was \$13.12. Find the rate of discount.

(8) I sold to a customer a bill of goods amounting to \$670. Which is the better for me, and by how much, to take his note for the amount for 90 days at 6%, and discount it at the bank at 8% the day that it is made, or to give him a 2% discount for cash?

(9) A man was offered \$400 cash for a team of horses, or a 6 months note (days of grace included) for \$450 without interest. If he chose the latter and discounted it at the bank at 8%, how much more than the cash offer did he receive?

(10) John Kelly bought \$800 worth of goods from the Symington Co., and gave in payment his note for 70 days, bearing interest at $7\frac{1}{2}$%. Ten days after receiving the note the company took it to the bank and had it discounted at 9%. How much did the Symington Co. receive for the note?

(11) A note is drawn June 11, at 60 days, with interest at 6%. It is discounted July 4 at 8%. Find (a) what fraction is the interest of the face value, (b) what fraction is the discount of the amount of the note, (c) what fraction is the discount of the proceeds.

(12) A merchant buys 240 yds. of silk at \$2.50 per yard and sells the silk at once at \$3 per yard, receiving in payment a 70 day note bearing interest at 6% per annum. which he immediately discounts at the bank at 8%. Find his profit.

Exercise XIVc

(Use the Simple Interest Tables, pp. 294-301)

(1) Find the simple interest on the following sums at 8% per annum:

(a) $640 for 15 days.
(b) $3411 for 30 days.
(c) $341.10 for 32 days.
(d) $34,110 for 34 days.
(e) $630.25 for 61 days.
(f) $1440 for 63 days.
(g) $983.51 for 90 days.

(2) Find the simple interest on the following sums at 6% per annum:

(a) $2150 for 18 days.
(b) $752.80 for 31 days.
(c) $730 for 33 days.
(d) $2993.68 for 60 days
(e) $1460 for 62 days.
(f) $11,802.70 for 64 days.
(g) $652.90 for 93 days.

(3) Find the amounts of the following notes, allowing interest at the rate of 6% per annum:

(a) Note for $500, drawn 11 Oct., 1924, at 15 days.
(b) Note for $1268, drawn 14 July, 1924, at 30 days.
(c) Note for $5585, drawn 6 Feb., 1925, at 1 month.
(d) Note for $5585, drawn 6 June, 1923, at 1 month.
(e) Note for $5585, drawn 6 Feb., 1924, at 1 month.
(f) Note for $5585, drawn 6 July, 1920, at 1 month.
(g) Note for $591.62, drawn 1 July, 1921, at 60 days.
(h) Note for $5862, drawn 18 June, 1922, at 90 days.

(4) Find the proceeds of the following notes, allowing discount at the rate of 8% per annum:

(a) Note for \$1000, discounted 32 days before due.

(b) Note for \$2100, discounted 62 days before due.

(c) Note for \$3420, discounted 90 days before due.

(d) Note for \$6871, discounted 15 days before due.

(e) Note for \$872.35, discounted 61 days before due.

(5) Find the proceeds of the following notes:

(a) Note for \$1500, drawn at 60 days, bearing interest at 6% per annum, discounted the day it is made at 8% per annum.

(b) Note for \$2460, drawn at 30 days, bearing interest at 6%, discounted 2 days later at 8%.

(c) Note for \$2920, drawn at 90 days. interest 6%, discounted 29 days later at 8%.

(d) Note for \$5200, drawn 18th June, at 90 days, bearing interest at 8% per annum, discounted 19th August, at 8%.

(6) A note for \$16,450 is drawn by the Cummings Knitting Company, 15th Sept., 1924, at 60 days, bearing interest at 6% per annum. It is discounted at the bank by the company the day it is made, at 6%. (1) What sum does the Knitting Company receive from the bank? (2) What sum does the bank collect from the Knitting Company when the note matures?

CHAPTER XV

PARTNERSHIP, STOCKS AND BONDS

A. Partnership

Two or more individuals may agree to conduct an enterprise to which each contributes certain values, such as capital, real estate, personal services, or anything that might prove of worth to the business. The profits or losses may be shared in any manner agreed upon by the partners. Values other than capital may be considered the equivalent of so much capital, or personal services may be paid for out of the funds of the firm as wages, or salary. After paying all expenses, including wages, interest, etc., the profits are divided among the partners. We shall assume, unless otherwise stated, that the net gain or loss is divided among the partners, in sums proportional to the investments with which they are credited.

NOTE.—Legally partners share profits and losses equally without regard to investment unless otherwise stated in their partnership agreement.

Example:

A and B form a partnership, each contributing $1800. In 3 months they admit C who furnishes $2000 capital. 3 months later B withdraws $400 and 2 months after that A puts in $300 more. How should a gain of $753.75 be divided at the end of the year.

A's investment is $1800 for 8 months
=an investment of $14,400 for 1 month
+$2100 for 4 months
=an investment of $8,400 for 1 month
Total $22,800 for 1 month

B's investment is \$1800 for 6 months
= an investment of \$10,800 for 1 month
+\$1400 for 6 months
= an investment of \$ 8,400 for 1 month

Total \$19,200 for 1 month

C's investment is \$2000 for 9 months
= an investment of \$18,000 for 1 month.

Total investment—A \$22,800
B \$19,200
C \$18,000 = \$60,000.

A gets $\frac{22800}{60000}$ of gain $= \frac{228}{600}$ of \$753.75 = \$286.42$\frac{1}{2}$
B gets $\frac{192}{600}$ of \$753.75 = \$241.20
C gets $\frac{180}{600}$ of \$753.75 = \$226.12$\frac{1}{2}$
Check Total \$753.75

B. Stocks

A group of individuals wishing to carry on a joint enterprise may organize an association and have it "incorporated," or given permission by the government to carry on the business as an individual carries on a business. The association is then known as a *corporation*, or *joint stock company*. Most of our large enterprises are carried on by joint stock companies. Each member of the company contributes whatever capital he may see fit and receives a certificate of ownership of a certain amount of stock, that is, of a certain number of *shares of stock*, corresponding to his contribution of capital. He is then a *shareholder* or *stockholder* of the company.

If the business is prosperous, and a gain is the result of its operation, after paying all operating expenses, interest, etc., this gain is divided at regular intervals,

usually once, twice or four times a year, among the stockholders. If there are 4000 shares altogether and the gain is $16,000, each shareholder is entitled to $4 for every share he holds. These payments are called *dividends*. The more prosperous the business, the larger will be the dividends.

Sometimes two kinds of stock certificates are issued: *preferred* and *common*. The holder of preferred stock has a prior claim on the net gain and hence has greater security, but his dividends are generally fixed at a certain amount per share. The holder of common stock does not get dividends unless there is more than enough gain to pay the dividends on the preferred stock. In that way his claim is less secure but there may be enough gain to pay him a dividend as large as that paid on the preference stock, or even larger.

Each share of stock has a nominal money value, in the majority of cases, $100. This is rarely its actual value, but purports to be equal to the amount of capital advanced when the share was issued. This nominal value is called the *par* value of the stock. When the $100 share of stock is worth $100 on the market, the stock sells at *par*. If the dividends are high and the security good the shares may sell at more than $100 each. The stock is then selling above par, or at a premium. When a $100 share sells at $115 the stock is said to be at a premium of 15%. Adverse conditions may cause the $100 share to sell at $85. It is then 15 below par, or at a discount of 15%. If on a share ($100) a dividend of $5 is received in a year, the stock is said to pay 5%.

The buying and selling of stock is usually done by brokers, who make a business of it and have access to the stock exchanges, or market places. If the broker

gets $\$\frac{1}{8}$, for buying or selling a \$100 share, this charge is called brokerage, and quoted at $\frac{1}{8}\%$. The brokerage on each \$100 share remains the same no matter what the market value of the share may be.

Suppose A sells 20 shares of stock (\$100 par value) at \$132, brokerage $\frac{1}{8}\%$, and B buys them. (Difficulties regarding the par value of the stock need not occur if the shares of stock are considered as 20 articles of merchandise).

A receives $\$132-\$\frac{1}{8}$ for each share,	
The 20 shares net him $\$131.87\frac{1}{2}\times 20$	\$2,637.50
The broker gets for selling $\$\frac{1}{8}\times 20$	2.50
B pays $\$132+\$\frac{1}{8}$ for each share,	
The 20 shares cost him $\$132.12\frac{1}{2}\times 20$	2,642.50
Of this the broker gets for buying,	
$\$\frac{1}{8}\times 20$	2.50

Both seller and buyer pay brokerage.

If the stock pays $5\frac{1}{2}\%$, B will now receive $5\frac{1}{2}\times 20=$ \$110 per year while he holds the stock. Since the par value of the stock is $\$100\times 20$ the amount of stock sold is given as \$2,000.

The market values of stocks constantly fluctuate. To an investor who buys stock so as to obtain a permanent income from his capital, these fluctuations are of small moment so long as the dividends are assured. Speculators, however, buy stock in the expectation that its market value will rise shortly and they can sell it again at a profit. The market value may go down, and if the speculator is forced to sell before it rises again, he loses. The transactions are often carried on, on *margins*. A speculator buys 100 shares of stock at 112 on a 10 point margin. He pays down \$10 a share and borrows

the balance from the broker or on the broker's security, leaving the stock certificates as collateral. He pays down $\$10 \times 100 = \$1,000$, and borrows $\$(112-10) \times 100 = \$10,200$. If the stock goes up to 117 and he sells at that price, he gains \$5 a share, less brokerage and interest on loan. If the stock goes down to 107, the broker will probably sell the stock to protect himself, and the speculator loses \$500 + interest, etc. If the stock is not sold he must advance more money, to cover the decline in value and preserve the 10 point margin.

A number of stock brokers may expect a certain stock to decline in value. They decide to accelerate the decline and make money out of it. They help the decline by selling "short." That is, they sell stock that they do not yet own. These increased offerings help to depress the market value. If the price goes down they buy at the lower price to "cover," that is, they buy enough at the lower market value to provide them with the stock they have already sold at the higher figure and make the difference. These brokers are known as "bears" with regard to this particular stock. Again, brokers expecting a rise in a certain stock, help the movement by buying freely and if the price goes up they sell at the higher price and make the advance. These buyers are "long" on the stock in question. That is, they buy stock they have no desire to hold, or have no guaranteed market for. They are known as "bulls" on the stock market.

If the shares of the stock are \$50 par value all quotations should be divided by 2. If the stock is selling at 80, the buying price of a \$50 share is \$40. If the stock pays 6% one share brings in a dividend of \$3 a year. If the brokerage is $\frac{1}{8}\%$ the broker gets $\$\frac{1}{16}$ for buying or selling a share. Sometimes, however, the price of a

$50 share is quoted instead of the price of two shares ($100 stock). If the shares are $200 each, quotations are doubled to get the dividend or brokerage per share. The price quoted may refer to the share or to the half-share ($100 stock).

Stock is sometimes issued, bought and sold, without being divided into shares. All quotations in this case are on $100 stock. The calculations are made in the same way as though the stock was issued in $100 shares, but transactions occur in amounts which would not constitute whole numbers of shares.

To each stockholder in a company there may be issued new stock to the extent of a certain fraction or multiple of the stock he already holds. This new stock may be issued without any extra investment of capital on the part of the stockholders. The stock of the company is then said to be *watered.* Since the amount of stock on which dividends are declared has been increased, the dividend rate is decreased. If the new stock is equal in amount to the previous issue the amount of stock is doubled and the dividend declared will be at one-half the previous rate for the same net profit. The rate of gain on the actual investment may thus be much greater than appears to be the case judging by the percentage dividend declared.

C. Bonds and Debentures

Bonds are written agreements to pay certain sums of money on definite dates, with interest at a stated rate per cent in the meantime. They are issued by governments, municipalities or joint stock companies, as repayments for loans. Unlike common stock, bonds call for fixed dividends, although their market values may change.

An improvement in the financial standing of the corporation issuing the bonds tends to raise their market value, as does also a decrease in current interest rates on money. Contrary conditions tend to cause a lowering of the market value.

Bonds are equivalent to, or are supported by, mortgages on the assets of a corporation or joint stock company. Interest on bonds issued by a joint stock company, and principal when due, have to be paid whether there is money to pay stock dividends or not. If bonds are registered in the name of their purchasers in the books of the corporation the interest may be remitted directly to those holding the bonds. Whether registered or not, they may have coupons attached, one for each payment of interest. These coupons are cut off by the holders as they come due and redeemed in cash by the corporation. Bonds (particularly of municipalities) are sometimes called debentures.

Provision for the payment of interest on bonds must be made from year to year, and a fixed amount yearly is set aside for the payment of the principal sum when due. The fund thus accumulating may be deposited in the bank or invested in reliable securities and is known as a *sinking fund.* It should be sufficient to retire the bonds when due.

Municipalities and School Boards often issue bonds requiring the payment of a portion of the principal sum each year. Such bonds may be *serial bonds* or *annuity bonds.*

When serial bonds are issued, they mature, not all at the end of a certain period, as in sinking fund bonds, but bonds representing a fixed fraction of the principal sum

mature each year. For example, if serial bonds for \$300,000 are issued, running 30 years, bonds to the face value of \$10,000 mature each year. In this case the annual payment made by the municipality or school board decreases yearly by an amount equal to the interest on \$10,000, owing to this decrease in the bonds outstanding, on which interest is payable.

When annuity bonds are issued, they are paid, principal and interest together, in equal, annual, instalments, running for a certain period of years. There is neither increase or decrease in the amount paid from year to year and at the end of the period the bonds are paid up principal and interest.

Bonds are bought and sold through brokers just as stock shares are bought and sold, and the brokerage is computed in the same way. A great part of the present National Debt of Canada has been incurred by borrowing money on bonds secured by the assets of the Dominion. War Loan or Victory Bonds pay 5 or $5\frac{1}{2}\%$ interest on their par value and are listed on the stock exchanges. Victory Bond coupons may be cashed at any chartered bank.

All quotations refer to bonds for \$100 each although bonds are issued in denominations both smaller and larger than this.

Examples:

(1) A man sells 450 shares of 5% stock at $96\frac{1}{8}$ and invests the proceeds in 6% stock at $134\frac{7}{8}$, brokerage $\frac{1}{8}\%$ in each case. How much 6% stock does he buy? How is his income affected?

He sells 450 shares for $\$96 \times 450$.

This buys $\dfrac{96 \times 450}{135}$ new shares (\$100) = 320 shares

$$\begin{array}{lr} \text{First income } \$5 \times 450 = & \$2250 \\ \text{New income } \$6 \times 320 = & 1920 \\ \hline \text{Decrease} & \$330 \end{array}$$

(2) What sum must be invested in $5\frac{1}{2}\%$ Victory Bonds selling at $103\frac{3}{4}$, to give a yearly income of \$6050, brokerage $\frac{1}{8}\%$?

$\$5\frac{1}{2}$ is the income from investing $\$103\frac{7}{8}$

\$6050 is the income from investing $\dfrac{6050}{5\frac{1}{2}} \times \$103\frac{7}{8} =$ \$114,262.50

\$114,262.50 must be invested.

(3) What % is an investor making on his money if he invests it in 6% stock at $112\frac{1}{4}$, brokerage $\frac{1}{4}\%$?

$\$112\frac{1}{2}$ buys 1 share (\$100)

This gives a yearly dividend of \$6

On $\$112\frac{1}{2}$ dividend is \$6

On \$100 dividend is $\dfrac{6 \times 100}{112\frac{1}{2}} = \$5\frac{1}{3}$.

He makes $5\frac{1}{3}\%$ on his investment.

(4) A hardware company has a capitalization of \$40,000, \$30,000 in common stock, and \$10,000 in preferred stock paying 6% dividends. A profit of \$3,000 is made in a year and distributed as dividends. What per cent dividend do the holders of the common stock receive?

Preference dividend = \$6 on each of 100 shares = \$600.

Common stock holders receive \$2400
\$30,000 is equivalent to 300 shares of \$100 each
On 300 shares dividend is \$2400
On 1 share dividend is \$8.
Common stock holders receive 8%.

(5) A speculator buys 1000 shares R.R. stock at 102, advancing the brokerage and \$5 a share and borrowing the rest of the cost at 8% per annum. In 30 days he advances \$15 a share additional, but is forced to sell immediately after at 85. What is his loss? Brokerage $\frac{1}{4}$%.

Cost $= \$102\frac{1}{4} \times 1000 =$	\$102,250.00
8% on \$97000 for 30 days	637.81
Total	\$102,887.81
Proceeds $= \$84\frac{3}{4} \times 1000$	84,750.00
Loss	\$ 18,137.81

Exercise XVa

(1) Three partners invest \$8,000, \$12,000 and \$20,000 respectively, in a hardware business. Their net profit in one year is \$9,640. Find the share of each in this profit.

(2) A, B and C form a partnership, A furnishing $\frac{1}{4}$ of the capital, B, $\frac{2}{5}$, and C the remainder. Find each partner's share of a net profit of \$5,600 made in one year.

(3) Brown, Jones and Smith enter into partnership to work a farm investing and sharing equally. 500 bushels of seed are required. Brown has no seed but Jones furnishes 300 bushels and Smith 200 bushels, Brown paying them \$200 for his share. Divide the \$200 between Jones and Smith.

(4) Gray invests \$7,200 in the firm of Gray and Thompson. Gray's first year's profit was \$1600 while Thompson's profit was \$1800. How much capital did Thompson invest?

(5) Two partner's interests in a business were in the ratio of 2 to 3. During a certain year their net profits were \$8,500. Their net worth at the end of this year was \$19,000. What was the condition of each partner's account at the beginning of the year?

(6) A, B and C enter into partnership and their gains are to be apportioned in the ratio of 2 : 3 : 4. Their total investment is \$18,000. At dissolution of partnership they have \$9,500 worth of goods, \$13,500 on deposit in the bank, and \$1,100 worth of collectable accounts on their books. How shall each share in the net gain?

(7) Brown starts business at the beginning of the year with a capital of \$4,800. On May 1st, he admits Jones as a partner with a capital of \$3,600. The net profits at the end of the year amount to \$2,373. Find each partner's share.

(8) Three people join in business, one putting in \$3,750 for two months, one \$6,500 for four months and one \$2,750 for six months. How should they divide a profit of \$4,200?

(9) A, B, C and D form a syndicate. A puts in \$2,500 for 30 days, B \$2,000 for 30 days, C \$1,000 for 28 days and D \$2,000 for 21 days. They make a profit of \$4,100. Assign the respective shares to each.

(10) Two partners have interests in a business in the ratio of 1 to 3. The net gain for the first year is 20% of their original investment. At the end of the year the

present worth of the firm is $24,000. Find the net gain of each.

(11) Wilson and Stevens formed a partnership and during the year made a profit of $2,556. At the beginning of the year Wilson put into the business $2,500 and at the end of 2 months $1,200. Stevens put in $3,000 at first and at the end of 3 months $800. How should they divide the year's profits?

(12) A, B and C form a partnership. On January 1, A invests $3,000, and on July 1, $2,000. On January 1, B invests $4,000, on May 1, $2,000 and on September 1, $1,000. On January 1, C invests $8,000, on March 1, $3,000 and on August 1, withdraws $4,000. How should the year's profits of $13,320 be divided?

(13) Brown and Reid form a co-partnership in business for 2 years. Brown at first contributed $3,000 to the joint capital and at the end of 12 months put in $1,500 more. Reid at first put in $3,500 but at the end of 15 months withdrew $1,000. At the beginning of the second year they admit Miller into the firm, he contributing $2,250. How should their profit of $6,240 be apportioned?

(14) Johnston and Wilson are partners. Johnston invests $1,200 for 5 months and then increases his investment to $2,200 for 5 months. Wilson invests $2,000 for 6 months and then withdraws $500. At the end of 10 months they dissolve partnership, having resources amounting to $5,730. What is each partner's share of the resources?

(15) A and B engaged in business with a capital of $12,000, of which A contributed two-thirds and B one-third. At the end of the first year A made an additional

investment of $7,000 and B increased his investment to $9,000. At the end of the second year the partnership was dissolved, and after all resources were converted into cash and all liabilities paid, the amount of cash in bank was $35,050. What was each partner's share of this amount?

(16) Brown, Jones and Smith took a contract to grade a section of a railroad. Brown furnished 35 teams for 40 days, Jones 25 teams for 50 days, and Smith 30 teams for 25 days. If the contract price of the work was $20,400, find what each should receive, and the amount earned per team per day.

(17) A and B are in partnership for a year and make a profit of $6,000, A's capital is to B's as 3 is to 4. At the end of 8 months A withdraws $\frac{1}{3}$ of his capital and B $\frac{1}{2}$ of his. How should the year's profits be divided?

(18) On January 1, 1923, Neal, Colter and Barton formed a partnership for two years under the name of Neal, Colter & Co., Barton being a silent partner. Neal was to have a salary of $2,400 and Colter $2,000. No salaries were to be drawn and no profits divided until the expiration of the two years. Neal invested $8,000, Colter $10,000, and Barton $6,000. On January 1, 1924 each partner increased his investment to $12,000. Find what each partner is worth on December 31, 1924, if the total resources were $74,240 and the liabilities other than salaries amounted to $11,340.

Exercise XVb—Oral

(Shares are $100 shares unless otherwise defined.)

(1) How many $100 shares in $500,000 stock?

(2) How many $500 bonds in a $20,000 bond issue?

(3) How many \$100 shares of stock will \$24,000 buy at par? at 96? at 120? at 75?

(4) What are the proceeds from the sale of 25 shares of stock at par? at 120? at 80? at 110?

(5) What is the yearly income from 80 shares of 5% stock? 8% stock? $5\frac{1}{2}$% stock?

(6) How many shares of stock will give a yearly income of \$450, if the stock pays 10%? 9%? 6%? $4\frac{1}{2}$%?

(7) What does the broker get for buying 60 shares of stock, brokerage $\frac{1}{8}$%? $\frac{1}{4}$%? $\frac{1}{2}$%?

(8) What does the broker get for selling \$40,000 stock, brokerage $\frac{1}{4}$%? $\frac{1}{2}$%?

(9) Find the cost of 40 shares C.P.R. stock quoted at $129\frac{3}{4}$, brokerage $\frac{1}{4}$%.

(10) Find the proceeds from the sale of 15 shares Bank of Commerce stock quoted at $160\frac{1}{4}$, brokerage $\frac{1}{4}$%.

(11) A man buys 1500 shares of stock at $88\frac{1}{2}$, and sells immediately at 90, brokerage $\frac{1}{4}$% in each transaction. What is his gain?

(12) A sells through a broker, 100 shares of stock to B when it is quoted at \$$112\frac{3}{8}$, brokerage $\frac{1}{8}$%. What does he realize? What does B pay? What does the broker get?

(13) What per cent does a man realize on his investment by buying 6% stock at par? at 75? at 120?

(14) I realize 5% on my investment by buying 6% stock. What do I pay for it?

(15) If 4% stock pays 5% on the investment, what is it worth?

(16) A company's capitalization is $4,000,000 preferred 6% stock and $6,000,000 common stock. What net profit will pay 4% to the common stock holders?

Exercise XVc

(1) I pay a broker $\frac{1}{8}$% commission to buy 9 shares of Northern Pacific R.R. stock worth on the market $103\frac{1}{2}$, what will be the total cost of the stock?

(2) Find the annual income from 326 shares of stock which pays a semi-annual dividend of $3\frac{1}{2}$%.

(3) What rate of interest on money invested shall I receive if I buy bonds at $53\frac{1}{2}$, which pay 4% interest, brokerage $\frac{1}{8}$%?

(4) Find the amount realized from the sale of nine $500 Province of Alberta bonds at $113\frac{1}{4}$, brokerage $\frac{1}{8}$%.

(5) What is the profit in buying 26 shares of Standard Oil stock at 150 and selling for 159, brokerage on each transaction $\frac{1}{8}$%?

(6) A man buys 200 shares of American Copper stock at 63, paying 4%, and holds it 2 years, receiving two dividends, and sells at 78, brokerage on each transaction $\frac{1}{8}$%. How much did he gain, if money is worth 6% per annum?

(7) Which is the best investment: 5% bonds at $82\frac{1}{2}$, 7% stock at $127\frac{1}{2}$, or money loaned at 6%?

(8) A man bought Illinois Central R. R. stock at $137\frac{1}{8}$, and sold it for $142\frac{1}{2}$, making a profit of $512.50. How many shares did he buy, brokerage $\frac{1}{8}$% in each transaction?

(9) If you can buy 10 shares of stock at 140 and get an 8% dividend each year, how much more income will you have yearly than if you loan the same money at 5%?

(10) What is the price of stock when

(a) 8% stock pays 5% on the investment?

(b) $5\frac{1}{2}$% stock pays 8% on the investment?

(c) $7\frac{1}{2}$% stock pays 10% on the investment?

(11) A person buys 120 shares of railway stock at $83\frac{3}{8}$, and sells six months later at $85\frac{1}{8}$, having received a dividend of 2%, brokerage $\frac{1}{8}$ in each transaction. At the time of making this investment, he could have loaned the money for six months with interest at 8% per annum. By how much is one a better investment than the other?

(12) A man received $1520 as the annual 8% dividend on stock that he owned. He then sold 80 shares at $142\frac{1}{4}$ and the remainder at 143, brokerage $\frac{1}{8}$ on each sale. What were the net proceeds of the sales?

(13) A broker owns 400 shares of stock which pays annual dividends of 7%. He sells at $140\frac{1}{2}$ and buys with the proceeds stock at $105\frac{3}{8}$, paying 6% dividends. How much is his income increased or decreased? What is the percentage increase or decrease?

(14) Find the cost, including brokerage, of 600 shares of stock paying 8% dividends if the rate of interest on the investment is $4\frac{1}{2}$%.

(15) A broker has an opportunity to buy 200 shares of 6% stock at a price to yield 8% on the investment, or, stock paying $2\frac{1}{2}$% semi-annually, at $62\frac{1}{2}$%, requiring the same investment. Which would be the better invest-

ment for him and by how much a year, if money is worth 6% per annum?

(16) A man's income consists of salary of $3000 and dividends from $12,000 of $6\frac{1}{4}$% stock. He sold the stock at $107\frac{1}{2}$ and bought $5\frac{1}{2}$% Victory Bonds, exempt from taxation, at $97\frac{1}{4}$, brokerage $\frac{1}{4}$ in each transaction. If his income tax was 4% on all incomes (not exempt) in excess of $2000, find the change in his net income.

(17) I bought 7% stock at $78\frac{1}{8}$, and after receiving a half year's dividend sold out at $79\frac{3}{8}$, paying $\frac{1}{8}$% brokerage on each transaction. If my capital was increased $292.50, how much did I invest?

(18) A man sells, through his broker, $5000 of 6% stock at $124\frac{1}{4}$, and invests in 5% stock, brokerage in each transaction being $\frac{1}{4}$. If his annual income is unchanged by this transfer, find the quotation for the latter stock.

(19) How many Province of Alberta $100 bonds must I sell at $107\frac{2}{5}$ to raise sufficient money to pay a debt of $3432.80, brokerage $\frac{1}{8}$?

(20) A man had $16,000 to invest. He invested one half of it in stock at $159\frac{7}{8}$, brokerage $\frac{1}{8}$, paying a quarterly dividend of $3\frac{1}{2}$%. The remainder he invested in city property yielding a monthly rental of $80, insurance and repairs costing him $\frac{1}{12}$ of the rent, and taxes 45 mills on 80% of the cost of the property. If his income is subject to a tax of 25 mills, find his total annual net income.

(21) What must be the rate of dividend on Great Western stock quoted at $44\frac{1}{2}$ in order that an investment in this stock may yield 6%?

(22) On June 15, 1922, the Calgary School Board borrowed by debenture \$210,000 to build and equip a school, the principal sum to be paid back in 20 equal annual instalments with interest at 6% per annum. Find the amount of the debenture payment the Board makes on June 15, 1925. What is the amount of the debenture falling due on June 15, 1939?

(23) A Railroad Company has a capital stock of \$50,000,000. During a certain year its gross earnings were \$20,250,000 and its operating expenses were \$15,400,000. The company declared an annual dividend of 8% and carried the balance of the net earnings to surplus fund. How much was this balance?

(24) A broker bought 300 shares of Union Pacific stock at 146. The market price of this stock declined to 140 and then rallied to $143\frac{1}{2}$. Thinking that another decline was coming, he sold 500 shares (200 short) at $143\frac{1}{2}$. The price continued to rally, however, and he covered by buying 200 shares at 145. What was his net loss on the whole transaction?

(25) On June 14, 1924, R. Davis sent a margin of \$900 to his broker to buy 50 shares of Canadian Pacific stock. The broker filled the order at $154\frac{7}{8}$, and on July 31 sold the stock at $172\frac{1}{4}$, charging $\frac{1}{8}$% brokerage each way and interest at 6% per annum. How much money should the broker remit to Davis?

(26) If I buy stock at 10% discount and sell at 10% premium, what per cent profit will I make on the investment?

(27) The capital stock of a company is \$1,200,000, one third of which is preferred stock, entitled to an 8% dividend, and the rest common. The net earnings for a

year amount to \$108,000 and are distributed in dividends. What rate of dividend is paid on the common stock?

(28) The net earnings of a railway company would justify a dividend of 6% if all the shares ranked equally. But, as \$2,000,000 of the stock consists of preference shares which receive $7\frac{1}{2}$% per annnum, the common stockholders receive only 5%. What is the whole amount of stock?

(29) The stock of a hardware company is \$200,000, half preferred and half common. After a 6% dividend is paid on the preferred stock, the rest of the net profit is divided, 25% to the preferred stockholders and 75% to the common stockholders. If the business for a year shows a profit of \$22,000, what rate of dividend is declared on each kind of stock?

CHAPTER XVI

OTHER APPLICATIONS OF SIMPLE INTEREST

A. Partial Payments

When an obligation is liquidated by an interest bearing note it is sometimes agreed between the maker and the payee that payments on the note will be accepted from time to time. The note may be made payable on demand or on some definite date.

When partial payments are accepted no reduction in the principal sum is made until all interest is paid up to date. If the payment upon any date is exactly equal to the interest due upon that date the payment simply cancels the interest accrued and interest on the principal sum begins again to accumulate from this date. If the payment is more than the interest due, whatever remains after paying the interest is applied to the reduction of the principal sum. If the payment is less than the interest due it is not applied in any way until the next payment is made, when the two payments are added together as though both were made on the date of the latter payment. The maker of the note would be just as far ahead if he withheld the small payment until such time as he might be able to pay the accrued interest.

Notes given to banks are generally drawn for short terms and when due if not paid in full are replaced by new short term notes.

Examples:

(1) On a demand note for $1100, dated 11th May, 1922, the following payments are made: 14th Feb., 1923,

$211.50; 20th April, $12.98. How much is due 14th May, 1923, allowing interest at the rate of 8% per annum?

Face of Note	$1100.00
Int. on $1100 for 95 days, 11th May to 14th Feb.	22.90
Due 14th Feb.	$1122.90
Paid 14th Feb.	211.50
Balance	$ 911.40
Int. on $911.40 for 65 days, 14th Feb. to 20th April	12.98
Due 20th April	$ 924.38
Payment (= Accrued Interest)	12.98
Balance	$ 911.40
Int. on $911.40 for 24 days, 20th April to 14th May	4.79
Due 14th May	$ 916.19

(2) On a note for $1625, dated 15th April 1923, at 6 months, bearing interest at the rate of 7% per annum are endorsed the payments:

1st July, 1923, $20; 15th August, $1400. How much is due 18th October, 1923?

Face of note	$1625.00
Int. on $1625 for 77 days 15th April to 1st July	24.00
Int. on $1625 for 45 days 1st July to 15th August	14.02
Due 15th August	$1663.02
Payment 15th Aug. ($20+$1400)	1420.00
Balance	$ 243.02
Int. on $243.02 for 64 days 15th Aug. to 18th Oct.	2.98
Due 18th October	$ 246.00

Since the payment of 1st July is not enough to pay the $24.00 interest due, the $24.00 is not added to the $1625, and the interest for the next period is calculated on $1625. The same result would be obtained if the interest were calculated on $1625 for 122 days and added as one sum

B. Average Term of Credit

A merchant buys $1000 worth of goods on 1st September. He may buy for cash or on a certain term of credit. If he pays cash, the $1000 belonging to the wholesale firm as the equivalent of the goods bought, is paid at once. If he buys on one month's credit he may use the $1000 for one month in his business before paying for the goods. If the credit term is two months he gets two months' use of the money before paying it over. If he buys $1000 worth on one month's credit and $5000 worth on three months' credit he can use the $1000 for one month and the $5000 for three months. The amount he owes is $6000. Instead of having the use of the $1000 for one month and the $5000 for three months it may be arranged that he have the use of the $6000 for a period longer than one month but shorter than three months before making payment. Finding when the whole $6000 should be paid in one payment giving the merchant the use of the $6000 that would be equivalent to the use of the separate sums is *averaging the terms of credit,* or finding the *equated time* of the two payments. The process of averaging the terms of credit is known as *equation of payments.*

Use of \$1000 for 1 mo. = use of \$ 1000 for 1 mo.
Use of \$5000 for 3 mo. = use of \$15000 for 1 mo.

Use of \$6000 for x mo. = use of \$16000 for 1 mo.

$$x = \frac{16000}{6000} = 2\tfrac{2}{3}.$$

Average term of credit $= 2\tfrac{2}{3}$ mo.

If the goods are bought on different dates some date must be selected from which all periods involved must be calculated.

Example:

A merchant bought \$500 worth of goods on 11th Aug. on 30 days' credit, \$1000 worth 1st Sept., terms cash; \$1500 worth on 1st Sept. on 60 days credit. When should the whole \$3000 be paid if all paid in one payment?

Evidently the first payment due is the \$1000 on 1st Sept. The next is \$500 due 30 days after 11th Aug., which is 9 days after 1st Sept. The next is \$1500 due 60 days after 1st Sept. 1st Sept. is a convenient date from which to reckon.

Use of \$1000 for 0 days = use of \$ 0 for 1 day
Use of \$ 500 for 9 days = use of \$4500 for 1 day
Use of \$1500 for 60 days = use of \$90000 for 1 day

Use of \$3000 for x days = use of \$94500 for 1 day

$$x = \frac{94500}{3000} = 31\tfrac{1}{2}$$

$31\tfrac{1}{2}$ days counts as 32 days.

Equated time is 32 days after 1 Sept. = 3rd October.

Time could have been reckoned from a date prior to 1st Sept. giving the same result, the numbers involved being somewhat larger.

If payment be not made upon the date determined by the average term of credit, interest on the whole sum should be calculated from this date.

Exercise XVIa

(1) 2000 Vancouver, Oct. 10, 1923.

Two years after date I promise to pay to the order of G. W. Kemp Two Thousand Dollars, with interest at 8% per annum. Value received.

Thomas Holden.

The following payments were endorsed on the above note: Dec. 22, 1923, \$800; May 6, 1924, \$250. What was the balance due at maturity?

(2) On June 15, 1922, a man bought a house for \$6500, paying \$5000 in cash, and giving a note for the remainder at 6%, with the privilege of paying all or part of it any time within two years. The following payments were made and endorsed on the note: September 8, 1922, \$300; Jan. 11, 1924, \$650. What amount was due on June 15, 1924?

(3) On June 30, 1921, a man purchased property for \$4000. He paid 40% in cash and agreed to pay interest half-yearly at 8% per annum on the balance with the privilege of paying off any part of this balance at the time of the interest payments. He made the following payments:

\$400 at the end of the first half year

\$600 at the end of the second half year

\$250 at the end of the third half year.

What sum on June 30, 1923, will cancel the debt?

(4) Find the balance due at maturity on each of the following notes, payments having been endorsed as indicated:

	Date	Time to run	Face	Int.	Endorsements
(a)	July 1, 1922	2 yr.	$3000	8%	Dec. 5, 1922, $500 June 25, 1923, $150 Jan. 11, 1924, $1250
(b)	May 7, 1920	3 yr.	$5000	6%	Sep. 30, 1920, $1500 Dec. 14, 1921, $2000 May 7, 1922, $ 750
(c)	Sept. 10, 1921	1 yr. 6 mo.	$2000	7%	Feb. 3, 1922, $ 800 May 10, 1922, $ 20 July 15, 1922, $580

(5) A demand note drawn Aug. 13, 1922, for $1000, bearing interest at 8% per annum, has endorsed on it the following payments:

Sept. 20, 1922, $75.60

Nov. 17, 1922, $90.00

Mar. 20, 1923, $10.00

What sum is required to cancel the note on June 30, 1923?

(6) On June 1, 1920, a man bought a farm for $6000. He paid $3000 in cash and gave a note for the remainder bearing interest at 6%. He made a payment of $500 on each of the following dates: June 1, 1921; June 1,

1923; Sept. 1, 1923; and April 1, 1924. What sum on June 1, 1924, will cancel the note?

(7) On Jan. 1, 1923, Jones owes Brown $650, on which Jones agrees to pay 6% interest and make a payment of $75 every three months and the balance on Jan. 1, 1924. When the balance is due Jones is unable to make any payment and Brown charges him 8% interest until paid. If Jones pays $85 on Feb. 1, 1924, and $135 on July 10, 1924, how much money must he raise to pay the balance Sept. 20, 1924?

(8) I paid a note that had been running just 5 years. It drew 8% interest, and the face was $900. It was drawn on Oct. 11, 1919, and $500 had been paid on it 2 years after date. What sum cancelled it?

(9) A note is settled 4 years after date. Its face value is $400 and it draws 8% interest. It has $150 paid on it 1 year and 8 months before final settlement. How much is required to settle the note?

(10) On April 6th, 1922, a man bought a farm for $7500, paying $5000 cash and giving a note for the remainder bearing interest at 6% per annum, with the privilege of paying all or any part of it at any time. The following payments were made and were endorsed on the note by the payee: Oct. 6, 1922, $500; Dec. 6, 1923, $50; Oct. 6, 1924, $1000. What amount was due on April 6th, 1925?

Exercise XVIb

(1-6, ORAL)

(1) How long will it take $10 to produce the same interest as $20 for 20 days?

(2) If I have the use of \$100 of Brown's money for 30 days, how much of my money should he have the use of for 10 days in return for the accommodation?

(3) A owes B \$200. \$100 is due in 2 months and the balance in 4 months. In how many months may the whole sum be equitably paid?

(4) The interest on \$60 for 2 months together with the interest on \$60 for 4 months is equal to the interest on \$120 for how many months?

(5) On June 1, I incurred a debt of \$300, \$200 of which is due in 3 months and the balance in 6 months. In how many months may the whole debt be equitably paid?

(6) If I owe \$100 due May 1, and \$500 due May 31, when may both accounts be equitably paid in one sum?

(7) \$300 is due in 2 months, \$250 in 4 months, and \$200 in 5 months; find the equated time of payment.

(8) Find the equated time of payment of \$8000 of which one-half is due in 3 months, one-quarter in 6 months and the balance in 8 months.

(9) A man owes \$780 to be paid in 8 months. He pays \$300 at the end of 4 months and \$260 at the end of 6 months. When will the remainder be due?

(10) On what date may the following items be paid in one amount without loss of interest to either party?

June 3, 1924, Mdse., 30 days, \$300
June 12, 1924, Mdse., 1 month, \$800
June 20, 1924, Mdse., 60 days, \$900
July 5, 1924, Mdse., 1 month, \$200

(11) A man owes the following: \$500 due in 3 months, \$600 due in 6 months, and \$1000 due in one year. If he pays \$750 immediately, when may the remainder be equitably paid?

(12) Find when the following bills may be equitably paid in one amount:

Sept.	1,	Mdse., 1 month	\$800
Sept.	15,	Mdse., 10 days	\$500
Oct.	1,	Mdse., 30 days	\$1000
Oct.	15,	Mdse., net	\$400

CHAPTER XVII

COMPOUND INTEREST AND PRESENT WORTH

A. Compound Interest Without Tables

If $1000 be deposited in a savings bank paying 3% interest on deposits, and the interest be not drawn out from year to year, it will be added to the principal sum and itself bear interest. Thus the depositor receives interest on interest. Suppose the bank in this case adds the accrued interest yearly. The amount on deposit will increase as follows:

Principal the first year	$1000.00
Interest the first year	30.00
Principal the second year	$1030.00
Interest the second year	30.90
Principal the third year	$1060.90
Interest the third year	31.83
Principal the fourth year	$1092.73

Or the amount in three years is $1092.73.

The compound interest on $1000 for 3 years at 3% per annum is $1092.73 − 1000 = $92.73.

Examples:

(1) Find the amount of $2400 in 5 years, allowing interest at 5% per annum, interest added yearly:

Principal	\$2400.00
Interest	120.00
New Principal	2520.00
Interest	126.00
New Principal	2646.00
Interest	132.30
New Principal	2778.30
Interest	138.915
New Principal	2917.215
Interest	145.861
Amount	\$3063.08

To take 5% of \$2400, multiply by 5 and move the product 2 places to the right. Moving 2 places to the right divides by 100.

Three digits beyond the decimal point, taking the third digit nearest the true product, as in contracted multiplication, will give an accurate result

(2) Find the amount of \$1000 in 2 yrs. and 3 months, allowing interest at 8% per annum, interest added half-yearly.

Principal	\$1000.00
Int. 1st $\frac{1}{2}$ yr.	40.00
	1040.00
Int. 2nd $\frac{1}{2}$ yr.	41.60
Amount in 1 yr.	1081.60
Int. 3rd $\frac{1}{2}$ yr.	43.264
	1124.864
Int. 4th $\frac{1}{2}$ yr.	44.995
Amount in 2 yrs.	1169.859
Int. 3 months	23.397
	\$1193.256

8% per annum would be 4% per half year and 2% for 3 months.

Amount in 2 yrs. and 3 months = \$1193.26.

B. Compound Interest with Tables

The interest on any sum of money in one year at 5% is $\frac{5}{100}$ of the sum. The amount at the end of the year is $\frac{105}{100}$ of the sum.

Taking the sum as	. . . . .		\$400
The interest is $\frac{5}{100}$ of 400	. . .	=	20
The amount is	. . . .		\$420

The same result could be obtained in one operation by taking $\frac{105}{100}$ of $400 = \$420$. If the \$420 is allowed to bear interest for the next year it will amount to $\frac{105}{100}$ of \$420, or \$441, at the end of the second year. At the end of the third year the amount will be $\frac{105}{100}$ of 441.

Putting all these operations together we have the amount of \$400 in 3 years $= \frac{105}{100}$ of $\frac{105}{100}$ of $\frac{105}{100}$ of $\$400 = \$400(\frac{105}{100})^3 = 400(1.05)^3$

Similarly the amount of \$620.50 in 7 years at 6% per annum, interest added yearly will be $\$620.50(1.06)^7$. If the interest be added half-yearly the amount will be $\$620.50(1.03)^{14}$, since the 3% for the half year has to be added 14 times in 7 years. So also the amount of \$1 in 5 years at 4% added yearly will be $\$1 \times (1.04)^5 = \$(1.04)^5$.

This formula enables us to construct tables which do away with the somewhat long process of adding interest term by term. It may also be used with advantage if tables are not available.

To find the amount of \$320 in 6 years at 5% per annum, interest added half yearly.

Amount $= \$320(1.025)^{12}$

Consulting the tables pp. 302–3, since $(1.025)^{12}$ is the amount of \$1 for 12 periods at $2\frac{1}{2}\%$ per period, select the column with $2\frac{1}{2}\%$ at the top, follow this down until opposite the number 12 at the left hand, the value of $(1.025)^{12}$ is given as 1.34489. The column at the left

gives the number of periods, which may be years, half-years or quarter-years if interest is added to the principal at these intervals.

Required amount = \$320(1.34489) = \$430.36

Examples:

(1) Find the amount of \$560 in 3 years, 10 months, at 6% per annum, interest added half-yearly.

Amount in $3\frac{1}{2}$ yrs. = $\$560(1.03)^7$

Rate for 4 months is 2%

Amount in $3\frac{1}{2}$ yrs. + 4 months = $\$560(1.03)^7(1.02)$

= \$560(1.22987)(1.02) = \$688.7272(1.02) = \$702.50.

(2) Find the amount of \$1200 in 7 years, 58 days, at 8% per annum, interest added yearly.

Amount in 7 yrs. = $\$1200(1.08)^7$ = \$1200 (1.71382)	= \$2056.58
Int. for 58 days on \$2056.58	= 26.14
Required amount	\$2082.72

(3) What rate per cent half-yearly, interest added half-yearly, is equivalent to 5% per annum, interest added yearly?

Let r be the rate % per half year.

Amount of \$1 in 2 half years = $\$\left(1+\frac{r}{100}\right)^2$

Hence $\left(1+\frac{r}{100}\right)^2 = 1.05. \quad 1+\frac{r}{100} = \sqrt{1.05} = 1.0247$

$\frac{r}{100} = .0247 \qquad r = 2.47$

The rate is 2.47% per half year.

(4) In what time will \$120 amount to \$262.93 at 4% per annum, interest added yearly?

\$120 amounts to \$262.93

\$1 amounts to 2.1911

This is $(1.04)^{20}$. approximately (see tables)

Time is 20 years.

C. Present Worth

The *present worth* of a sum of money payable some time in the future is the sum, which, if put at interest *now*, would amount to the aforementioned sum at the given time in the future, allowing interest at whatever rate money is supposed to be worth. The present worth of \$150 due 1 year hence, if money is worth 6%, is the sum which in 1 year at 6% would amount to \$150.

Solution: In 1 year \$1 would amount to \$1.06;

Conversely, \$1.06 is amount of \$1 in 1 year at 6%

$$\$150 \text{ is amount of } \frac{\$150}{1.06} = \$141.51 \text{ in 1 year.}$$

\$141.51 is the present worth, or present value of \$150 due in 1 year. The difference between \$150 and \$141.51, its present value, is the *true discount* on \$150 due in 1 year. The true discount is the interest on \$141.51, the present worth.

In true discount, *interest* is allowed at the rate named. In bank discount, the interest allowed is slightly greater than the rate named, for short periods, as shown in treating of Bank Discount. For long periods the difference

would be considerably greater. In dealing with accounts and all ordinary negotiable paper, bank discount is practically always used. In dealing with bonds, debentures, mortgages, annuities and large money payments extending over considerable periods of time, the method of present worth is used to determine the cash value of payments due in the future, and compound interest is allowed.

Examples:

(1) Find the present value of \$1150 due in 2 years, if money is worth 7% per annum, and interest added half-yearly.

\$1 in 2 years would amount to $\$(1.035)^4$

Conversely,

$\$(1.035)^4$ due in 2 yrs. has present value of \$1

\$1 due in 2 yrs. has present value of $\dfrac{\$1}{(1.035)^4}$

\$1150 due in 2 yrs. has present value of $\dfrac{\$1150}{(1.035)^4}$

$=\dfrac{\$1150}{1.14752}=\$1002.16$, by contracted division.

Instead of dividing \$1150 by 1.14752, if the value of $\dfrac{1}{(1.035)^4}$ is expressed as a decimal, multiplication can be used instead of division.

$$\frac{1}{(1.035)^4}=\frac{1}{1.14752}=.87144$$

$$\$1150(.87144)=\$1002.16.$$

As stated already $\frac{1}{(1.035)^4}$ is the present worth of \$1 due in 4 half years, allowing interest at $3\frac{1}{2}\%$ per half year.

The value of $\frac{1}{(1.035)^4}$ may be found on p. 304. Use the column headed $3\frac{1}{2}\%$ and opposite 4 (on the left) will be found .87144.

Note:

True Discount = Interest on present worth,

Bank Discount = Interest on the sum,

= Interest on (present worth + true discount),

= Interest on present worth + interest on true discount,

Bank Discount = True Discount + interest on true discount.

or, Bank Discount − True Discount = Interest on True Discount.

Exercise XVIIa

(1) What is the compound interest on \$2000 for 3 years at 6%, compounded annually?

(2) What will \$1000 amount to in 2 years at 8% per annum, interest compounded half-yearly?

(3) A boy 18 years of age has \$1500 deposited in the savings bank for him. The bank pays 4% interest, compounded semi-annually. What amount will he have to his credit when he is 21 years of age, no withdrawals nor any more deposits having been made?

(4) At the beginning of each year for 5 years a railroad company put aside out of the profits of the preceding year $40,000 as a sinking fund. If these sums had been invested at 5%, compound interest, compounded annually, what would the amount have been at the end of the tenth year?

(5) The simple interest on a sum of money for 3 years at 6% is $450. What is the compound interest on the same sum for the same time and rate?

(6) If a man deposits $3000 in a savings bank, which pays 4% per annum, compound interest, compounded semi-annually, and leaves it on deposit for 2 years, 6 months, will he have more or less money than if he had loaned the $3000 at simple interest for the same time at $4\frac{1}{2}$%? How much more or less?

(7) The difference between the compound interest and the simple interest on a sum of money for 2 years at 6% is $10.80. Find the sum.

(8) A man despoits $500 in the Savings Branch of a bank on January 1, 1921, $700 on January 1, 1922, and $400 on July 1, 1922. If the interest allowed is 4% per annum, compounded half-yearly, find what sum will be standing to his credit in the bank on Dec. 31, 1922.

(9) By what fraction of the principal does the compound interest exceed the simple interest for 3 years at 5%?

(10) $600 amounts to $841.53 in 5 years at interest compounded yearly. What is the rate per cent. per annum?

(11) The compound interest on a certain sum for the second year is $108, and for the third year is $116.64. Find the rate and the principal.

(12) What sum must be invested at the beginning of each year to amount to $300 at the end of two years, if the interest earned is at the rate of 6% per annum, compounded half-yearly?

(13) A person borrows $500. At the end of each year he pays $125 to reduce the principal and also to pay interest at 8% on the sum which has been standing against him during the year. Find how much he will owe after three payments.

(14) 6% per annum compounded half-yearly is equivalent to what rate per cent. per year?

(15) What rate per cent. per half year, compounded half-yearly, is equivalent to 8% per annum, compounded yearly?

(16) A man accumulates money to buy a house by saving $800 each year for 4 years. He deposits this sum at the end of each year with a Trust Company which pays him 4% per annum compounded half-yearly. The house which he wishes to purchase is valued at $4000. How much more money will he need? $(1.02)^8$

(17) A man's salary is $3000 per year. On Jan. 1 of each year 5% of the preceding year's salary is placed in a pension fund which bears compound interest at 4% per annum. To what sum will his contributions amount at the end of 5 years from the date of the first deduction, including payment due?

(18) A city sets aside each year $20,000 to provide a sinking fund which, at 4% compound interest, will be sufficient to redeem 20 year municipal bonds at maturity. What will be the amount of the sinking fund at the end of four years?

(19) A man takes out a life insurance policy for $3000 at the age of 20 and pays the single premium of 408.30 per $1000. He dies at the age of 45 and the heirs receive the face of the policy and $921.50 profits. How much better off are they than they would have been had the money been deposited in a savings bank paying $3\frac{1}{2}\%$, compounded yearly?

(20) A merchant increases his capital by 20% each year. The increase during the past two years was $6600. What is his present capital?

(21) A man 40 years old sends to the government $2560 to purchase an annuity to begin when he reaches 65 years of age. He dies at the age of 60 and the original payment, with interest at 4% per annum, compounded yearly, is paid to the heirs. What is the amount of this payment?

Exercise XVIIb

(1) A man owes a debt of $2500 due in 8 months. How much should he pay his creditor to-day, so that if put out at interest at 6%, the payment will amount to the face of the debt at maturity?

(2) Find the present worth of:

(*a*) $125.40, due in 1 year, money being worth $4\frac{1}{2}\%$.

(*b*) $1000.00, due in 3 years, money being worth 6%.

(*c*) $3500.00, due in $3\frac{1}{2}$ years, money being worth 4%.

(*d*) $2600.00, due in 2 years, money being worth 5%.

(3) A owes B $400, payable at the end of 10 months, $600 payable at the end of 8 months, and $1000 payable at the end of one year. If money is worth 6%, find what sum paid now would discharge this total debt.

(4) What payment to-day will cancel a debt of $720, due in 10 months, if money is worth 6%?

(5) A merchant's cash price for an article is $150 and his 3 months' credit price is $155. If money is worth 8%, which is the better price for the buyer?

(6) In the purchase of a house, the buyer is to pay $1000 at the end of each year for three years. If money is worth 8% per annum, what is the equivalent cash value of the house?

(7) If $250 is the present worth of $255, due in 4 months, what is the rate of interest?

(8) What sum will amount to $2812.16 in 3 years at 4%, compounded yearly?

(9) A offers for a house $2100 payable at the end of three years; B offers $455 cash and $455 at the end of each year for three years; C offers $1600 cash. Money being worth 8%, which of these offers is the best and how much better is it than the others?

(10) The interest on a sum of money for a year is $47 and the true discount on the same sum due in one year is $45. What is the sum?

(11) A father wishes to give his son $5000 on his twenty-first birthday. What sum should be invest at 5%, compound interest, on the son's sixteenth birthday to provide for this amount?

(12) The present worth of $268.66 due in a certain number of days is $252.50. If the rate of interest is 8%, find the number of days.

(13) A borrows from B a sum of money and agrees to pay him by three annual payments of $500 each. If

money is worth 6% per annum, compound interest, find the sum borrowed.

(14) A mortgage of $1500 has three years to run. It bears interest at the rate of 6% per annum, payable yearly, the next interest being due one year from now. An investor who considers money worth 8% per annum purchases this mortgage. How much did he pay for it?

(15) A person buys bonds paying 6% yearly, which are to be paid off at par 3 years after the time of the purchase. If money is worth 5%, compound interest, what price should he pay for a bond?

(16) A father deposits in the savings bank a sum of money for each of his three children whose ages are respectively 14 yrs., 16 yrs., and 18 yrs., so that each child shall receive $2000 when he or she reaches the age of 21 years. If the bank pays 4% compound interest, what sum does he deposit for each child?

CHAPTER XVIII

EXCHANGE

A. Domestic Exchange

It is constantly necessary to make payment for some commodity at a more or less distant point in Canada, or to send money to some other centre in the Dominion for investment purposes. The practice of sending small sums in cash or postage stamps, in registered letters is becoming less common, as other methods are considered safer. When large sums have to be sent from one financial institution to another they are carried to the required point in most cases by an express company which assumes the risk of loss in transit. In the vast majority of cases, however, the money is not sent at all. In place of the cash a *post office money order*, a *postal note*, a *personal cheque*, an *express money order*, or a *bank draft* is sent instead of the cash. When the cash is urgent a *telegraphic money order* may be sent.

When the person desirous of discharging a debt at some other place pays his money into the post office he receives in return a post office money order or postal note. This he sends to the person or firm that should receive the money and it is cashed on presentation by the proper person at the point desired. If he pays into an express office he receives an express money order which is used in the same manner. A bank draft may be purchased and sent in the same way. In a great many cases payments are made by personal cheque.

THE BANK OF NOVA SCOTIA

$ 300 $\frac{00}{100}$ No. 50

Regina, June 15th 1925

Pay to the order of Robert Thompson

Three Hundred $\frac{00}{100}$ Dollars.

and charge to this Bank

The Bank of Nova Scotia

Vancouver, B.C.

MANAGER

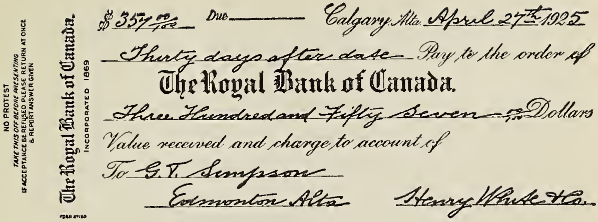
ACCOUNTANT

A. Bank Draft (1)

A bank draft is simply a cheque issued by one bank on another bank requesting the payment of a sum of money to a person or firm named, and in form is similar to the foregoing.

In case of accounts due or overdue the person or firm to whom payment is due may "draw on" the debtor. The draft in this case is sent, on instructions by the creditor, to a bank where the money is to be collected, and requests the debtor to pay the amount into that bank. (See Form 2). It is presented by this bank and the debtor is supposed to pay it if it is a "sight" draft, or to "accept" it if it is payable on a certain date in the future. If accepted it becomes a promissory note. If neither paid or accepted it is returned to the starting point and the debt remains unpaid.

The debtor "accepts" the draft by writing across its face the word "accepted," the date, and his signature.



NO PROTEST
TAKE THIS OFF BEFORE PRESENTING
IF ACCEPTANCE BE REFUSED PLEASE RETURN AT ONCE & REPORT ANSWER GIVEN
The Royal Bank of Canada.
INCORPORATED 1869

$357 00/100 Due——— Calgary, Alta. April 27th 1925

Thirty days after date Pay to the order of

The Royal Bank of Canada.

Three Hundred and Fifty Seven 00/100 Dollars

Value received and charge to account of

To G. T. Simpson
Edmonton Alta — Henry White Co.

(Form 2)

Bank Draft (2)

In nearly all the cases mentioned a sum slightly in excess of the payment to be made has to be paid by the sender. This excess is called *exchange*. It increases as the amount to be remitted becomes larger. In cases of drafts and cheques it is generally a percentage of the face of the draft or cheque, perhaps $\frac{1}{4}\%$, $\frac{1}{8}\%$ or $\frac{1}{10}\%$ of it. If the money is sent otherwise than through a bank the exchange must be paid at the sending office. If through a bank, it may be understood that it is to be paid either by the person sending or the person receiving. If the person receiving pays the exchange it is deducted from the face of the draft or cheque where presented for payment.

In some cases drafts on certain centres are in demand by the banks to obviate the necessity of having money forwarded from that centre. Drafts may then be purchased for their face value or even less. Exchange on these places is then at par, or at a discount, as the case may be.

B. Foreign Exchange—Direct

If money is due at some point in the United States the debt may be dischared by any of the means used in domestic exchange except the postal note. It is then practically a case of domestic exchange, except that the value of the Canadian dollar in terms of the American dollar varies slightly from time to time, and vice versa. This has been the case to a greater extent than usual since the war. If the Canadian dollar is at a discount it will require a sum in Canadian currency in excess of the amount to be paid in American currency to secure the required money order, draft, etc. If the Canadian dollar is at a premium the reverse will be the case. The causes of these fluctuations are quite complex. Two considerations, however, affecting them may be mentioned. The financial solvency of Canada tends to keep the Canadian dollar up at least to par value in terms of the United States currency and of all other currencies. The smaller the amount of money due from this country to the United States, compared to the amount due from the United States to Canada, that is, the more favorable the trade balance, the higher will be the quotation of Canadian currency in terms of American.

The expression of the value of the Canadian dollar in terms of the American dollar on any given date, or vice versa, is known as the *course of exchange* between this country and the United States on the date mentioned. The *par of exchange* between Canada and the United States is $1.00 Canadian currency = $1.00 American currency.

The course of exchange between different countries of the world since the war has been much disturbed. The par of exchange is quoted at the same rate as before but

the course of exchange has differed very greatly from the par relation.

Debts may be paid in foreign countries, or other British countries, by post office money order, but remittances are made for the most part by bank drafts, which in this case are known as *Bills of Exchange.* Variations in the course of exchange between Canada and other countries are due to the same causes as variations of Canadian, in terms of United States currency. The inconvenience of sending money, which means the transference of gold either as specie (coined gold) or bullion (uncoined gold), becomes greater the further the countries are separated, and variations in the course of exchange due to unequal trade balances are greater than for countries closer together.

The question of the financial solvency of the different countries with which we trade and their estimated ability to redeem their paper currency in gold is also a large factor in determining the course of exchange.

The par of exchange between countries with currencies based on a gold standard is arrived at by comparing the amounts of pure gold represented by their respective monetary units. In Great Britain 1869 sovereigns weigh 40 lbs. Troy and are $\frac{11}{12}$ pure gold. £1 then contains $\frac{40\times5760}{1869}\times\frac{11}{12}$ grains of pure gold. In Canada and the United States $10 (gold) weighs 25.8 grains and is $\frac{9}{10}$ pure, $10 then contains $\frac{9}{10}\times25.8$ grains of pure gold. Comparing these weights, £1 is worth $4.86656 which is generally considered equal to $$4.86\frac{2}{3}$.

The old par of exchange between Great Britain and Canada was fixed at £1 = $$4\frac{4}{9}$. The course of exchange

is sometimes quoted as a certain % premium on this old par. Sterling exchange quoted at $109\frac{1}{2}$, or simply at $9\frac{1}{2}$, means that £1 is worth $\$4\frac{4}{9} \times 109\frac{1}{2}/100 = \$4.86\frac{2}{3}$. The form will indicate whether the old or new par is used in the quotation. The par of exchange between Canada and France might be given as 1 fr. = \$.193, or it is sometimes quoted at 5.18, which means that \$1 is worth 5.18 francs. Similar variations occur in other cases.

C. Arbitrated Exchange—Indirect

The course of exchange between any two of the various countries carrying on world trade is a matter of financial knowledge available for our banks. Instead of sending bills of exchange directly to the country which the payment must eventually reach, the course adopted may be to send them to some other country first. Currency of this other country is purchased in this manner, and then new bills are sent to the desired centre. Two, or even more, countries may intervene before the final destination is reached. The general effect of the circuitous route is the avoidance of the necessity of sending gold from country to country and on account of this effect the indirect route may be cheaper than the direct.

The "Commercial Intelligence Journal" issued January 10, 1925, by the Department of Trade and Commerce gave the following as the closing quotations of the values of other currencies in sums of Canadian dollars for the week ending January 5, 1925, as furnished by the Foreign Exchange Department of the Bank of Montreal.

	Monetary	*Parity*	*Course of Exchange*
Britain	£	\$4.86	\$4.7775
France	Franc	.193	.0540
Italy	Lire	.193	.0420

Holland	Florin	.402	.4070
Belgium	Franc	.193	.0500
Spain	Peseta	.193	.1405
Portugal	Esc.	1.08	.0484
Switzerland	Franc	.193	.1956
Germany	Mark	.238	......
Greece	Drachma	.193	.0182
Norway	Crown	.268	.1525
Sweden	Crown	.268	.2716
Denmark	Crown	.268	.1774
Japan	Yen	.498	.3888
India	Rupee	2s.	.3612
United States	$	1.00	1.0034
Mexico	$	.498	.4916
Argentina	Peso	.424	.4038
Brazil	Milreis	.324	.1166
Roumania	Lei	.193	......
Jamaica	£	4.86	4.7826
Shanghai, China	Tael	.708	.7740
Batavia, Java	Guilder	.402	.4064
Singapore, Straits Settlements	$	.567	.5594
Most of British West Indies	$	1.00	.9733 to .9984

Examples:

(1) What is the cost at Vancouver of a draft on Toronto for $1152, exchange $\frac{1}{4}\%$ premium?

Face of Draft	$1152.00
Premium $\frac{1}{4}\%$	2.88
Cost	$1154.88

(2) If exchange on Paris is quoted at 1 fr. = \$.075, for how many francs will \$125 buy a bill of exchange payable in Paris?

\$.075 buys bill of exchange for 1 fr.

\$125 buys bill of exchange for $\frac{125}{.075} = \frac{125000}{75} = \frac{5000}{3} =$ 1666.67 fr.

(3) Find the cost of a bill of exchange on London for £420, exchange quoted at \$4.86$\frac{2}{3}$.

$$\text{Cost} = 420 \times 4.86\tfrac{2}{3} \text{ dollars} = \$\frac{420 \times 14.60}{3} = \$2044.00$$

(4) If sterling exchange is quoted at \$4.60, exchange on Paris 6.25 francs to the dollar, Paris on London 28.5 francs to the £., how much less will it cost a Montreal merchant for a bill of exchange on London via Paris for £212 10s. than direct on London?

Cost direct = \$212.5 × 4.60 = \$977.50

Cost via Paris = 212.5 × 28.5 francs =

$$\frac{\$212.5 \times 28.5}{6.65} = \$969.00$$

Advantage by Paris \$ 8.50

Exercise XVIII

(1) (*a*) Find the cost, in Calgary, of a draft for \$1440 on Toronto at $\frac{1}{4}\%$ premium.

(*b*) Find the cost in Toronto of a draft on Birmingham for £2500, exchange being quoted at \$4.84$\frac{1}{2}$.

(2) A commission merchant in Vancouver sold goods amounting to \$2400 on a commission of $1\frac{1}{2}\%$. He

remitted the proceeds by a sight draft purchased at $\frac{1}{4}\%$ premium. What was the face of the draft?

(3) How large a bill of exchange on London, England, can be bought for $5000, exchange being quoted at $4.85?

(4) Find the total cost of cabling £1200 to Liverpool, England, if exchange is quoted at $\$4.86\frac{2}{3}$, and the charges are, commission $\frac{1}{4}\%$ and cablegram of 16 words at 25¢ per word.

(5) (*a*) If the Canadian dollar is quoted at 4% discount in New York, what amount in Canadian currency will pay a debt of $500 in New York?

(*b*) A Canadian visiting San Francisco exchanged $750 in Canadian currency for American money. How much American money does he receive if the Canadian dollar is quoted at $\frac{1}{2}\%$ premium in San Francisco?

(6) The cost of a sight draft in Edmonton for $1750 on Vancouver is $\$1754.37\frac{1}{2}$. Find the rate of exchange.

(7) A merchant in Regina remits $3000 to London, England, exchange being at 109. What will be the face of his bill?

(8) A Canadian merchant buys a draft to pay an account of 1800 francs in Paris. If exchange is quoted at 5.18 francs to the dollar, find the cost of the draft.

(9) What are the proceeds of a bill of exchange on Manchester, England, fcr £320 4s. 6d. at 60 days after sight, if discounted at once at 6% per annum, exchange being at $\$4.87\frac{1}{2}$?

(10) If sterling exchange on demand is $4.8685, find the corresponding 60 day quotation, interest 6%.

(11) If in New York demand-bills are sold at $4.87½ and bills at 60 days' sight at $4.83½, what is the rate of discount (no days of grace allowed)?

(12) Find the total cost of travellers' cheques as follows: 30 cheques of $10 each; 15 at $25 each; 8 at $50 each; and 2 at $100 each, if the bank issuing them charges ½%.

(13) If sterling exchange is quoted at $4.86⅔, and exchange between London and Berlin is 20.3 marks on the pound, what should a Vancouver merchant pay for a bill of exchange on London to pay a debt of 4500 marks in Berlin?

(14) If sterling exchange is quoted at $4.865, and exchange between London and Paris is 25.10 francs to the pound, what should a Calgary merchant pay for a bill on London to pay a debt in Paris of 10,000 francs?

(15) Find the cost of a sight draft on New York for $2572.50, if the American dollar is quoted at 1.014.

(16) At 25¢ per word and 1% of the amount, find the cost of a forty-two word cable money order from Halifax to Paris for 35,000 fr. when exchange is quoted at 5.15 fr. to the dollar.

(17) A broker sold for me a bill on Liverpool, England, at $4.84½ and charged ⅛% brokerage. What was the face of the bill if the proceeds were $2609.25?

(18) An article which costs 7s. 6d. in London sells for $4.00 in Montreal. Find the profit made by an importer who buys 1000 of these articles in London and sells them in Montreal after paying 5d. per article for freight, etc., and a customs duty of 20% paid in Montreal on the

invoice price. Exchange at Montreal on London is 4.85.

(19) Will it pay to remit \$5600 from New York to Paris via London under the following quotations: Exchange at New York on London is \$4.86$\frac{1}{4}$; at London on Paris is 25.18 francs to the pound; and at New York on Paris 5.3 francs to the dollar? What is the difference in proceeds in francs between the direct and indirect routes?

(20) A Toronto merchant purchased goods invoiced at 8400 francs. What will it cost him for a bill of exchange on London to pay for the goods if sterling exchange is quoted at \$4.865, and 26.25 francs to the pound?

(21) My agent in Brussels, purchased for me a carpet costing 25,000 francs and charged a commission of 5%. I remitted him a draft to cover the cost of the carpet and commission. If the draft was paid by cheque and exchange was quoted at 5.15$\frac{5}{8}$ fr. to the dollar, what was the amount of my cheque?

REVIEW EXERCISE III—BUSINESS ARITHMETIC

(1) A mixture contains 19.8 grams of saltpetre and 13.2 grams charcoal. Calculate its percentage composition.

(2) A merchant buys flannel listed at $2 a yard, with discounts of 40%, 10% and 5%. What per cent. on cost does he gain by selling at $2 a yard?

(3) An agent sells 4100 lbs. of wool at 36 cents a lb. on a commission of 3%, and after taking out his commissions invests the remainder in sugar at $11.25 a cwt., on a commission of $2\frac{1}{2}$%. What is his total commission and how many lbs. of sugar does he buy?

(4) An owner insures a house worth $15,000 for 3 years, for $4000 in one company, paying a rate of $1.15, and for $8000 in another company at $1.05. If the house is destroyed by fire during the term of insurance, calculate the loss sustained by the owner and by each company.

(5) John Jones insures his life for $2000, paying a premium of $64.75 a year. In case of death after eight premiums have been paid, how much do the heirs receive above what has been paid in premiums?

(6) A farmer insures a wheat crop of 200 acres against hail for $20 an acre on a premium of 7%. Damage is estimated at 75% of the value owing to hail. What premium does he pay? How much insurance does he receive? If he sells the damaged crop for $12 an acre, how much does the crop net him?

(7) A village assessed at $450,000 requires $6975 for schools, $3375 for debentures, and $6300 for current expenses. Calculate the mill rate for each of these purposes and the total mill rate.

(8) A married man with four children under 18 years of age, has an income of $4900. His exemptions are

$2000, and $500 for each child. Calculate his income tax at 4%.

(9) A merchant buys 200 yards of silk in England at 10 shillings a yard. He pays an ad valorem duty of $22\frac{1}{2}$% and a specific duty of 10 cents a yard. He pays $17.50 freight. How much a yard does the silk cost him if exchange is quoted at $4.80?

(10) A and B invest $4000 each in a partnership business. At the end of 4 months A withdraws $1000 and 4 months later B puts in an extra $2000. How should a gain of $3500 be divided?

(11) A owns $5000 railway stock paying 5% dividends. He sells at $110\frac{1}{4}$ and invests the proceeds in municipal bonds at $99\frac{3}{4}$, brokerage $\frac{1}{4}$% in each case. If the bonds are 6% bonds, how much does he increase his annual income?

(12) A note for $2430 dated 11 June at 90 days is discounted on 3 July at 8%. Find the proceeds.

(13) On a demand note for $640, bearing 8% interest, dated 3rd Jan., 1922, are the following endorsements: 5th Feb. $116; 7th Mar. $200; 11th April $32.75. How much will settle the note on the first of May, 1922?

(14) A merchant buys on 3rd Sept. $420 worth of goods, terms cash; $310 worth, terms 30 days; $530 worth, terms 60 days. Find the average term of credit and the date on which the bill should be paid if paid in one payment.

(15) Find the amount of $550 in 5 years at 7% per annum, interest compounded yearly.

(16) Find the amount of $400 in 8 yrs. 9 mo., at 6% per annum, interest added half-yearly.

(17) Find the sum of money which, invested 1st Jan., 1924, bearing interest at the rate of 5% per annum,

compounded half-yearly, will amount to $1000 on 1st Jan., 1930.

(18) What will a draft on Toronto for $352.80 cost, exchange $\frac{1}{4}$% premium?

(19) Exchange on Paris is 17.5 francs to the dollar. Paris on London is 77.5 francs to the pound. What sum will purchase a bill of exchange on London, via Paris, for £586 10s.?

(20) A rectangle is measured and found to be 12 cm. long and 10 cm. wide. If the measurements are each 1 mm. too long, what is the percentage error in the area?

REVIEW EXERCISE IV—BUSINESS ARITHMETIC

(1) My agent in Winnipeg sold a consignment of grain and after taking out his commission of 2% and paying a freight bill of $2125, sent me a draft for $19,533. What was the selling price of the consignment?

(2) A furniture factory was supplied with an automatic sprinkling system. It saved the company $1248 a year insurance, which was a saving of 66$\frac{2}{3}$% on what the company formerly paid. What insurance do they now pay?

(3) When the cost of flour and the other ingredients of bread is 60% of the selling price, and the labor, overhead and distributing expenses 30% of the selling price, 10 loaves are sold for $1. The cost of flour, etc., dropped 25%; the other expenses per loaf remained unchanged. How many loaves may now be sold for $1 in order to make the same percentage of profit as before?

(4) By testing his wheat seed a farmer found that 8% of the grains did not germinate. By planting the selected seed his yield was 45 bushels per acre. What was the increased yield through testing the seed?

(5) At what price shall a dealer mark goods costing \$2.40 a yard so that he may make a profit of 10% when selling at a discount of $33\frac{1}{3}\%$ off the marked price?

(6) A fur coat cost \$240, less 30% and $16\frac{2}{3}\%$. It was marked \$200 by the retailer but in order to make a quick sale for cash he gave a 10% discount. What rate of profit did he make?

(7) A shoe merchant buys shoes at \$6 per pair, less $16\frac{2}{3}\%$ and 10%. The expenses for purchasing and freight amount to 15¢ per pair. At what price must he sell them to make 40% profit if it costs 25¢ per pair, on the average, to sell them?

(8) I can buy a living room suite from one firm for \$450, less $33\frac{1}{3}\%$ and 20%. I can buy the same suite from another firm for \$400, less 25% and $12\frac{1}{2}\%$. The terms are in each case net 30 days, 5% cash. What is the least amount of ready money for which I can get the suite?

(9) A man drew 60% of his money from the bank and with 40% of it bought a motor car for \$1800. How much money had he remaining in the bank?

(10) A merchant's selling price of an article is \$75 less 20%. If he buys at the same list price, \$75, what discount in addition to a discount of 25% must he allowed in order to clear a profit of $33\frac{1}{3}\%$?

(11) A man had \$1500 to make a cash payment on a house and lot which he purchased for \$4200. He borrowed the balance from a Loan Company, giving as security a mortgage on the house and lot. The house rented for \$45 a month. He paid 8% interest on the mortgage; 1.3% premium on an insurance policy of \$3000; a tax of 38 mills on the dollar on a valuation of

$3500, and $60 for repairs. What rate of interest did he receive on his investment?

(12) Clerks in R. Jones' department store are allowed 10% discount on all purchases. A clerk bought a hat for $7.20. The regular sale price of the hat was 20% reduction on the price the hat was originally marked. By selling the hat to the clerk the firm still made a profit of 20%. What did the hat cost the firm? Had the firm sold at the original marked price what would have been the per cent. of profit?

(13) If 15 pounds of a dry article has lost 10 oz. in drying, what per cent. of its original weight was water?

(14) A firm can take a contract for $175,000 and be reasonably certain to make 20%. In order to do this it must borrow $65,000 for 3 months at 8%. How much will the firm make by borrowing the money and taking the contract?

(15) A merchant buys bankrupt stock at 55 cents on the dollar and sells it at 10% below the original list price. What is his rate of profit?

(16) A man was offered $9000 for a house. He afterwards sold it for $6800, which was 15% less than it cost him. What per cent. would have been made if the first offer had been accepted?

(17) It costs $720 to manufacture a motor car. What must be the catalogue price of this car, so that the manufacturer can give the retailer 25% discount on the list price, pay an agent $16\frac{2}{3}$% commission on the selling price and still make a profit of 25%

(18) A merchant sent $12,170 to his agent with instructions to buy tea and prepay the freight. The agent's commission was 1%. The tea cost 60 cents a pound and

the freight was 25 cents per cwt. Find the number of pounds of tea purchased.

(19) A farmer shipped 2860 bushels of wheat to Vancouver, freight prepaid, requesting a commission agent, whose rate is $\frac{3}{8}\%$,to sell the wheat. The agent sold the wheat at \$1.72$\frac{1}{2}$ a bushel. The agent then, on request, purchased as many cattle at \$47.50 a head as the proceeds would pay for, charging \$2.50 a head commission, reserving to prepay the freight \$94.20 per car on four carloads of cattle. He sent the balance of the proceeds to the farmer. How many cattle did he buy and what balance did he send?

(20) R. Williams, gave up his lease on an apartment, renting at \$65 per month, bought a lot for \$1200 and built a house costing \$3800. The taxes on the property amount to \$120.75, repairs \$27.30, and cost of insurance \$19. If money is worth 8%, how much does Williams save in a year by owning his own house?

(21) An agent receives a consignment of 5000 pounds of tea which he sold at 41 cents per pound on a commission of 5%. He invests the proceeds in flour at \$4 per barrel, having deducted his commission at 2$\frac{1}{2}$% for buying. Find the number of barrels of flour bought, and his total commission.

(22) An agent sells apples at \$4.50 per barrel and buys sugar at 6 cents a pound with the proceeds after deducting his commissions. The rates of commission are 2% for selling and 1$\frac{1}{2}$% for buying. His total commission is \$63. How many barrels of apples did the agent sell?

(23) A person who derives equal parts of his income from three investments, yielding respectively 3%, 4% and

5%, consolidates them into a single investment yielding 4%. By how much per cent. has his income been increased or decreased?

(24) The three edges of a brick are measured with a faulty foot rule which makes the measurements appear 10% too small. What will be the percentage of error in the calculated volume?

(25) A shipment of 1600 lambs, average weight 75 pounds, was sent to my agent in Chicago. He sold the lambs at 10 cents a pound on a commission of 2%. He was instructed to use the proceeds of the sale to buy tea at 40 cents a pound, to prepay the freight on the tea, 20 cents per cwt., and retain his commission of 1% on the purchase. How many pounds of tea did he buy?

(26) How much water must be added to a barrel of vinegar 85% pure, to reduce it to a 50% solution?

(27) How much pure alcohol must be added to a gallon of alcohol, 20% pure, so that the mixture shall be 30% pure?

(28) If a dry article exposed to damp air absorbed 3 oz. of water, and then weighed 7 lbs., what per cent. of its present weight is water?

(29) The capital of a railway company is $20,000,000. It borrows on bonds $12,000,000 at 4% per annum Its gross receipts for the year amount to $3,000,000 and the operating expenses are 40% of the gross receipts. What dividend can be declared if $50,000 has to be placed aside for a reserve fund?

(30) A man has a net income of $2319 from a fixed salary, and the rent of a house. On the house which rents for $50 a month, there is a mortgage of $2000 at 6% per

annum, \$4000 insurance at $1\frac{1}{4}\%$ premium, taxes at the rate of 19 mills on the dollar on an assessment of \$5000, and on his salary a tax of 10 mills with \$400 exempt. What is his salary?

(31) The market quotation on apples for a certain day was \$3.65 a barrel. After receiving 275 barrels a falling market made it necessary for the commission firm to sell all of them at \$3.45. If the freight was 25 cents per barrel, draying 10 cents per barrel and commission 5%, what was the loss in net proceeds due to the change in the market?

(32) A grain dealer in Chicago ordered his agent at Duluth to buy 10,000 bushels of wheat, paying $2\frac{1}{2}\%$ for buying. The grain was shipped down the lakes, and a policy at $1\frac{1}{2}\%$ was taken out to cover the cost of the grain and commission. The cost of the insurance was \$184.50. At what price per bushel was the grain bought?

REVIEW EXERCISE V—BUSINESS ARITHMETIC

(1) Barbed wire was quoted by a wholesale firm at \$4.75 per spool. This price was subject to a discount of 7% if bought in carload lots, and a further discount of 5% off this latter price if paid in cash. A merchant bought a carload of 560 spools for cash, sending the wholesale firm a marked cheque for the amount, on which he paid the bank exchange of $\frac{1}{10}$ per cent. Freight charges amounted to \$80.00. He sold the wire to a rancher for \$5.25 per spool, receiving in payment a 70 day note, bearing interest at the rate of 8% per annum, which he discounted immediately at the bank at 10%. Find the merchant's net gain.

(2) On Oct. 1, a note payable in three months was drawn, bearing interest at 6% per annum. The note was immediately discounted at a bank at 8%. The difference between the interest and bank discount was $4.04. Find the face value of the note.

(3) A merchant imports 50 bales of canton flannel, each bale containing 100 yards. The flannel is 27 inches wide and invoiced at 20 cents a yard. The duty is 1 cent per square yard with an additional duty of 15% of the invoice price. The merchant insures it in transit for 80% of its value at a premium of 2.3%. Freight charges are $75.00. At what price per yard must the flannel be sold to make a clear profit of $33\frac{1}{3}$%?

(4) An importer paid for freight and duty an amount equal to 10% of his net cost price, and sold to the retailer at a profit of 20% of his gross cost. The retailer sold at 28% profit What was the net price to the importer of goods that the retailer sold for $5280?

(5) A builder contracted to build a house in 12 weeks and agreed to pay $25 a day or portion of a day that the work remained incomplete after the end of that period. To fulfil his contract he engaged 15 men to work 10 hours a day. At the end of three weeks 10 of the men struck and remained away from their work for a fortnight. After their return the number of working hours was reduced to nine. What was due from the builder for his breach of contract?

(6) The records of a business show gross sales amounting to $1,250,000 for the year. The cost of doing business for the year was $200,000, and the net profit was $300,000. What must be the selling price of articles that cost $300

in order to cover the cost of doing business and the profit?

(7) A farmer shipped wheat to a commission merchant in Winnipeg. The merchant sold the wheat at \$1.10 a bushel, retaining his commission of 3% and paying out \$50.80 freight charges. With the balance he purchased a draft, the bank charging $\frac{1}{10}$ per cent. The farmer received \$2081.12. How many bushels of wheat were sold?

(8) I bought an automobile in Chicago for \$840 and shipped it into Alberta, prepaying the freight, amounting to \$92. To cover loss during shipment I insured the automobile for $\frac{6}{7}$ of its value at a rate of $1\frac{7}{8}$%. The duty on automobiles entering Canada from the United States is $33\frac{1}{3}$% of the invoice price. During shipment, damage to the extent of \$160 was done to the automobile and the insurance company paid the claim. I received an offer of \$1000 for the car on arrival. If I accepted the offer would I gain or lose on the whole transaction, and how much?

(9) At an experimental station of an agricultural college it was found that soil which had been fertilized produced $7\frac{1}{2}$ tons to the acre while that which had not been fertilized produced only 4.6 tons to the acre. What was the per cent. of gain due to fertilization?

(10) On March 3, 1923, A. C. Cameron sold a quarter section of land at \$46.50 per acre. The purchaser gave a morgtage for 60% of the purchase price, \$1000 cash, and a 6 months' note bearing interest at 7% per annum for the balance. Cameron immediately discounted the note at the bank at 8% per annum and sold the mortgage at a

discount of 5% off face value. Find his net cash receipts from the completed transaction.

(11) In estimating the cost of painting a building, the contractor estimates that one gallon of paint will cover 250 square feet and that the labor will cost twice as much as the material. What should a contractor ask for painting a house two coats, the perimeter being 200 feet and the average height 20 feet, if paint costs him $3.60 per gallon and he wishes to make a 20% profit on material and labor?

(12) In a town municipality the amount of taxes remaining unpaid at the end of the year 1923 was $2815. To these arrears there was added on Jan. 1, 1924, a sum equal to 10% of them as a penalty for non-payment. To meet expenditures for the year 1924 the town will require $45,000. On the expectation that 90% of all taxes, arrears included, will have been paid on or before Dec. 31, 1924, what will be the rate of tax levy for the year on an assessment of $932,000?

(13) Land near the City of Edmonton produces on an average $150 worth of garden truck per acre at a total expense of $45 per acre. Land of similar quality but not so favorably located has for its most profitable crop, wheat, averaging 40 bushels per acre, the market value of which is $1.05 per bushel. The expense per acre for this crop is $9.50. Disregarding taxes, the land per acre in each case is drawing interest on what capital at 6%?

(14) A field produces on an average 750 lbs. of tobacco per acre, average sale price 12 cents per pound, at a cost of $45 per acre. This land is paying interest at 5% on what amount? Suppose the tobacco wilt to take possession of the field, rendering tobacco raising impossible,

and the next most valuable crop is corn, average yield 25 bushels per acre, market value 75 cents per bushel, and cost of raising \$9.50 per acre, what capital per acre at the same rate will the land represent?

(15) A broker sold for a farmer 10,000 pounds of pork at $8\frac{1}{2}$ cents a pound on a commission of $3\frac{1}{2}\%$. He paid \$19.55 for freight and invested the balance in lumber after deducting his commission of 2% for investing. How much lumber did he purchase if he paid \$40.00 per thousand feet?

(16) A person builds a house costing \$5000 on a lot worth \$2000, and rents to a tenant at \$70 a month. The lot increases in value 5% and the house depreciates 4% during the year. The house is insured for 80% of its cost at $\frac{5}{8}\%$ per annum. The taxes are at the rate of 36 mills on the dollar on an assessment which is 90% of the cost of the property. What rate per cent will he earn on his original investment?

(17) A merchant in London, England, consigns to his agent in Montreal 5000 yards of cloth invoiced at 6 shillings a yard, and instructs him to sell the cloth and invest the proceeds in apples, after deducting the following charges: Commission 5% for selling and 5% for buying, freight on cloth and apples, \$328.15. The cloth is sold at \$2.50 a yard and the apples are bought at \$3.50 a barrel. Find the merchant's profit if the apples net him in London 20 shillings a barrel.

(18) A Toronto importer bought 1800 yards Brussels carpet, $\frac{3}{4}$ of a yard wide, invoiced at £300. The specific duty is 28 cents per square yard, ad valorem duty 40%. Ocean freight and charges on this side amount to \$278.25.

At what price per yard must he sell the carpet to gain 20%? Sterling exchange quoted £1 = $4.865.

(19) The ice-man pays $5 per ton for ice and retails it at 50 cents per 100 lbs. If delivery costs $2 per ton and the loss is 20% by melting, what per cent profit does he make on the cost price?

(20) How many 12-ounce loaves of bread can be made from a barrel of flour (196 lbs.) if the weight of the bread is $133\frac{1}{3}$% of the weight of the flour used in making it? What would be the selling price of a barrel of flour at 5 cents a loaf for the bread?

(21) If when corn is cut and cured in the shock the loss in dry matter is 31%, and when put into the silo 11%, what is the total gain in dry matter by putting the corn from 10 acres, weighing 12 tons per acre, into a silo if, when ready for cutting, corn contains 73.2% of water? How much is the dried matter saved worth if silage containing 79.1% water has a feeding value of $3 per ton?

(22) If the farmer makes $12\frac{1}{2}$% profit on his wheat, and the miller charges 20% for grinding it into flour and the grocer makes a profit of 10% by selling a barrel of flour for $5.65, what did it cost the farmer to produce the wheat required to make a barrel of flour?

(23) A and B are engaged in different lines of business with a capital of $6000 each. The first year A gains and B loses 20% of the investment. The second year A loses and B gains 20% on the capital each has then. Which is now the better off?

(24) The Dominion Fire Insurance Company charges an average rate of $1\frac{1}{4}$% for insurance for a term of one year. Its agents are allowed a commission equal to 15%

of the premium. Expenses of management, office rent, printing, etc., amount to \$7,400 per year. Fire losses for the year equal 44% of the total premiums. What must be the total amount of risk in order that the company's receipts for a year may exceed its disbursements by \$4,900?

(25) A merchant settled with his creditors by paying them 48 cents on the dollar, and had \$3000 left. Had they seized his entire effects and sold them they would have received 56 cents on the dollar, after paying the costs, which would amount to $6\frac{2}{3}$% of the value of the effects. Find the full amount of the merchant's indebtedness.

(26) A dealer in Edmonton shipped 3 carloads, of 48,000 lbs. each of potatoes to an agent in Spokane, U.S.A., who sold them at \$2.25 per bushel, f.o.b. Spokane. The freight charges were 45 cents per cwt. and the agent charged 3% commission on the sale, both charges payable in Spokane. Find the net returns in Edmonton for the shipment if the American dollar is quoted at 1.014.

(27) A man in Winnipeg holds a Paris draft for 175,000 francs, which was drawn on June 5th at 3 months. How much should he receive for it on June 27th if he discounts it at 6% per annum, the rate of exchange being \$1 = 5.18 francs?

(28) Five-eighths of an investment of \$120,000 was raised by stock sold at par and the remainder by a bond issue bearing 6% per annum. The gross yearly return was \$53,000, of which 85% went to pay operating expenses. After meeting the interest charges on the bonds, what per cent per annum may be paid on the stock issue?

(29) A mortgage of \$1200 has three years to run. It bears interest at the rate of 8% per annum, payable

yearly, the next interest being due one year from now. An investor who considers money worth 6% is about to purchase this mortgage.

(*a*) How much should he pay for it?

(*b*) To obtain the money for this purchase the investor must sell shares of 3% stock, which he owns, at $101\frac{3}{4}$, brokerage $\frac{1}{8}$. How many shares must he sell?

(*c*) What change will he make in his annual income the next two years by purchasing the mortgage?

(30) A and B enter partnership, A investing $25,000 and B, $15,000. They agree that at the end of each year they shall draw from the profits 4% on the amounts they have invested as interest on capital, that B shall receive in addition 20% of total profits for acting as manager and that the remainder of the profits shall be divided equally between them. If in a year A received in all $3750. what did B receive?

(31) A grain dealer in Montreal instructed his agent at Calgary to buy 30,000 bushels of wheat at 95 cents per bushel, 15,000 bushels oats at 32 cents per bushel and 5.600 bushels of barley at 55 cents per bushel. He paid his agent 2% commission for buying. A policy at $1\frac{1}{2}$% was taken out to cover the cost of the grain and the commission. Find the amount of the policy and the premium paid.

CHAPTER XIX

GRAPHS

A. Graphic Representation of Statistics

For the purpose of rendering a comparison more easily comprehended various statistics are represented graphically. Numerous devices are used in graphic representation. Perhaps the simplest graph is the straight line or bar. Straight lines placed side by side, parallel, and the lengths proportional to certain magnitudes that are to be compared, give a clearer idea of the comparison than the numbers do.

A booklet issued by a Life Assurance Company makes the statement that of 1000 who enter public school in childhood, 343 enter high school, 72 enter college and 23 graduate. Represent this graphically.

1000 enter Public School

343 enter High School

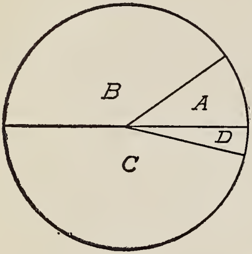
72 enter College

23 graduate.

Instead of straight lines vertical figures may be used, such as figures of men to represent comparative populations, of bags of flour to represent exports of flour by different countries, of ships to represent tonnage. Figures of other objects may be used for the same purpose. The heights of the figures, or their areas may be made proportional to the magnitudes to be compared. Such representations are sometimes called pictograms.

The circle is used for similar purposes. For example:

In a country containing 60 million people, 5 million are Christian, 25 million are Mohammedan, 28 million are Buddhist and 2 million are Jewish. Represent this graphically by areas of sectors of a circle.



A represents 5 million Christians

B represents 25 million Mohammedans

C represents 28 million Buddhists

D represents 2 million Jews.

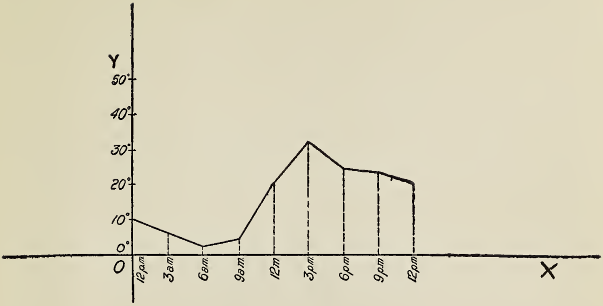
B. Graphs Connecting Two Variables

Squared Paper

Very often changes in a varying magnitude bear some definite relation to changes in some other magnitude. This relation is generally shown by representing units of one magnitude along a horizontal line, and units of the other magnitude along a vertical line cutting the former, a certain length of line representing the unit in each case. We may call these lines *axes* for convenience, the horizontal line the x-axis, and the vertical line the y-axis.

Example:

At midnight the thermometer reads 10°; at 3 a.m., 6°; at 6 a.m., 2°; at 9 a,m., 4°; at noon, 20°; at 3 p.m., 32°; at 6 p.m., 25°; at 9 p.m., 24°; and at midnight 20°. Represent this graphically.



From O, cut off along the x-axis equal distances, each distance to represent the lapse of 3 hours time. Along the y-axis let corresponding lengths represent 10° temperature. If O represents midnight then the successive points marked along Ox will represent 3 a.m., 6 a.m., 9 a.m., noon, etc. At midnight the temperature is 10°. The point marked 10 on the y-axis *vertically above* O shows this fact. From the point marked 3 a.m., measure vertically upward a distance representing 6°, that is, a distance $\frac{6}{10}$ as long as from O to the point marked 10°. From the point marked 6 a.m., measure upward a distance representing 2°. Do the same with the other points marked on Ox. Join the temperature points as shown. This broken line represents changes in temperature for 24 hours. If squared paper were used the measurements could be achieved by simply counting the small squares vertically and horizontally. The greater the number of temperature readings taken in the 24 hours, the more accurately will the changes be shown. A machine that draws a pen a certain horizontal distance every hour over paper marked with close horizontal lines, the whole

connected with a thermometric contrivance causing the pen to move upward or downward a definite distance with every degree rise or fall in temperature, would produce a temperature curve or chart recording accurately the temperature at all times.

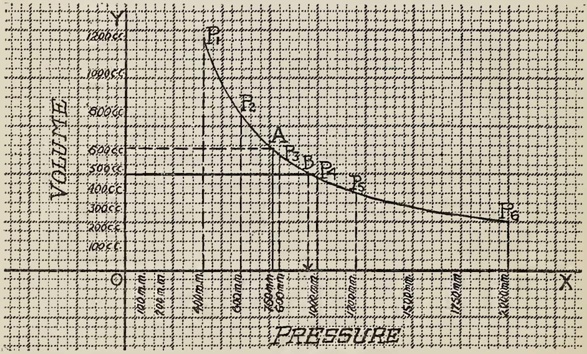
The figure constructed above answers the purpose of a rough temperature curve and shows the variations in temperature as related to lapse of time for 24 hours.

Examples:

(1) The volume of a given mass of gas under a pressure of 400 mm. of mercury is 1200 c.c.; under pressure 600 mm., 800 c.c.; pressure 800 mm., 600 c.c.; pressure 1000 mm., 480 c.c.; pressure 1200 mm., 400 c.c.; and pressure 2000 mm., 240 c.c. Represent this graphically.

What would be its volume when the pressure is 760 mm.?

What pressure would give a volume of 500 c.c.?



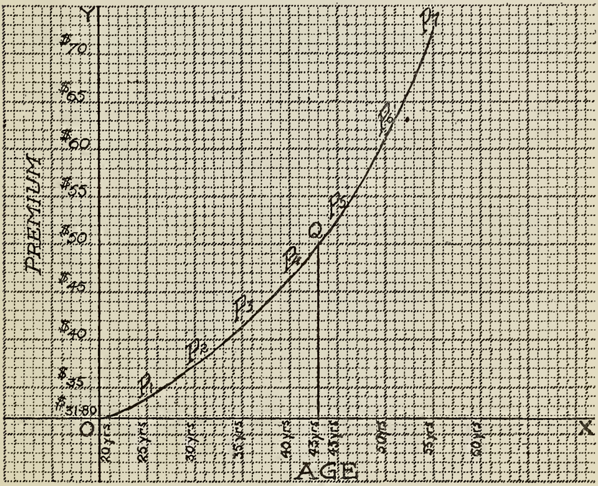
(Fig. 1.)

Let each small square on the x-axis (Fig. 1) represent 50 mm. pressure and each small square on the y-axis

50 c.c. volume. The data given fix the points P_1, P_2, P_3, P_4, P_5 and P_6, and so determine the graph between the limits of 400 mm., and 2000 mm., pressure.

When the pressure is 760 (M) the volume will be approximately 630 c.c. as shown by the position of the point *A*. When the volume is 500 c.c. the pressure will be approximately 960 mm., as shown by the position of *B*.

(2) Represent graphically the increase in annual premiums as the age at which insurance is taken rises from 20 years to 55 years for a 20 pay life policy, using



(Fig. 2.)

the Life Insurance table on page 164. What should the premium be for age 43?

From this curve (Fig. 2) the cost for age 43 would be approximately $50 a year, as shown by the point *P*.

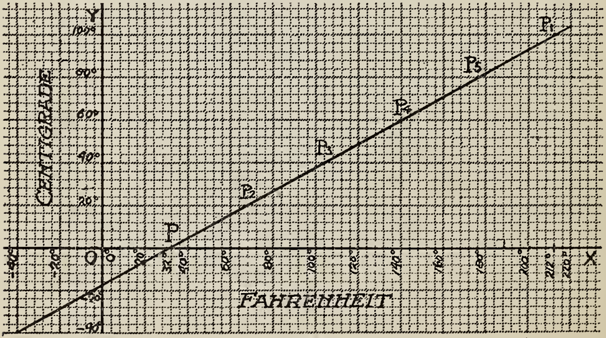
Approximations of the annual premium for ages between the limits, 20 years and 55 years, can be read from the curve. It should be observed that the vertical lines above Ox show the increase in the premiums but are not proportional to the premiums.

C. Comparison of Units by Graphs

A graph may be used to express approximately: metres in terms of yards, pounds in terms of kilograms, rods in terms of yards, temperature Centigrade in terms of temperature Fahrenheit, dollars in terms of francs or pounds sterling, pounds Troy in terms of pounds Avoirdupois, or conversely in each case. Many other relations may be similarly shown.

Example:

Represent graphically the relation between temperature Centigrade and temperature Fahrenheit.



(Fig. 3.)

Use the x-axis (Fig. 3) for Fahrenheit units of temperature and the y-axis for Centigrade units. Let O

represent zero in both scales and each small square represent 4°, as marked. Degrees below zero in each case would be properly represented by negative numbers. In the graph, measurements downward from the x-axis would be negative and represent degrees Centigrade below freezing point. Measurements to the left from the y-axis would be negative and represent "below zero" Fahrenheit.

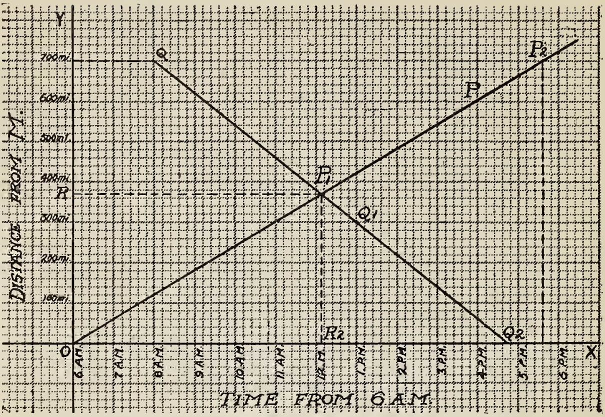
As O° Centigrade and 32° Fahrenheit are equivalent, the graph at 8 squares to the right of the y-axis (32°F) must be O units above Ox. That is, the graph cuts the x-axis at the point P. So also, 100°C. and 212°F. are equivalent, which gives another point P_1 on the graph. It can be readily shown that 68°F. and 20°C. are equivalent, also 104°F. and 40°C., etc. These give a series of points on the graph. They are all on the straight line PP_1. The straight line PP_1 is the graph required and the approximate equivalent Centigrade of any temperature expressed by the Fahrenheit scale may be read from it, and vice versa. Zero F. is almost −18°C.

D. Other Applications

The graph may be employed to show when and where two objects travelling in the same or opposite directions will pass each other.

Example:

An aeroplane leaves M for N at 6 a.m., travelling in a straight line at a uniform velocity of 60 miles an hour. Another leaves N for M at 8 a.m., travelling at 80 miles an hour. If it is 700 miles from M to N, show graphically when and where they meet and at what hour each arrives at its destination.



(Fig. 4.)

Let distance along Ox (Fig. 4) represent the number of hours elapsing, beginning at 6 a.m., as marked, and along Oy let each 5 squares represent 100 miles distance from M.

The first plane starts from M. O is on its graph. In 10 hours it goes 600 miles which gives the point P. OP is the graph showing its progress.

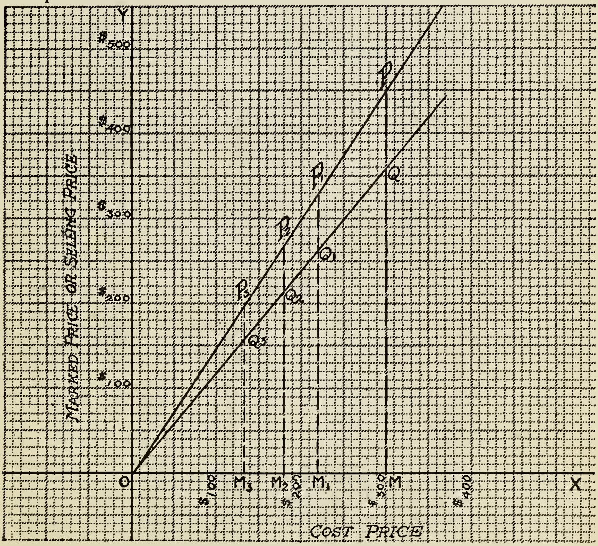
The second plane is 700 miles from M at 8 a.m. Q is on its graph. In 5 hours (at 1 p.m.) it has flown 400 miles towards M and is then 300 miles from that point. Q_1 is on its graph. QQ_1 is the graph showing its progress.

Let the graphs cut at P_1. This point shows that both planes were the same distance from M at the same time and so determines the point and hour of meeting. The place of meeting is about 368 miles from M and the time about 12.10 p.m.

The first graph cuts the 700 mile line at P_2, showing that the first plane's journey finished at about 5.40 p.m. The second graph cuts the x-axis at Q_2, showing that the second plane is 0 miles from M or has completed its journey at about 4.45 p.m.

Graphs give approximate results only, the accuracy of the approximation depending on the scale used in constructing the graph. If 10 small squares represent one unit, results can be read correct to the second decimal place. The relations connecting three varying quantities, such as cost price, marked price and selling price of goods sometimes admit of simple graphic representation.

Example:



(Fig. 5.)

A merchant marks his goods at 50% above cost price and allows a discount of 20% off the marked price. Show graphically the relation of the three prices, each to the others, for goods costing up to \$300. Show the other two prices when (*a*) cost is \$220, (*b*) marked price is \$270, (*c*) selling price is \$160.

Let cost price in dollars be represented by distance along Ox, \$10 by one small square. Let marked price be represented along Oy, using the same scale (Fig. 5). Goods costing \$300 are marked \$450. OP is the graph of the marked price. Goods marked at \$450 sell for \$360. OQ is the graph of the selling price.

(*a*) If the cost price is \$220, (M_1), the graph shows that the marked price is \$330 (P_1), and the selling price approximately \$264 (Q_1).

(*b*) If the marked price is \$270 (P_2), the cost price is seen to be \$180 (M_2), and the selling price \$216 (Q_2).

(*c*) If the selling price is \$160 (Q_3), the marked price is seen to be \$200 (P_3) and the cost price about \$133 (M_3).

Exercise XIXa

(1) The following gives in percentage form the world's supply of cotton contributed by different countries in one year:

United States	63%
India	16%
Egypt	7%
China	5%
Russia	4%
All others	5%

Represent the above data by a circular pictogram.

(2) The net income of a railway company for a certain year was apportioned as follows:

Preferred stock dividends	$3,000,000
Common stock dividends	12,000,000
Sinking Fund	400,000
Profit and Loss Account	3,600,000

Represent these statistics by a bar pictogram.

(3) A thermometer was placed in some water which was then heated, and the temperature was read every minute. Draw the temperature graph from the following table:

Time in minutes	0	1	2	3	4	5	6
Temperature in degrees	12	16	19.5	22.5	25	26.7	27

(4) The height of a barometer at noon, on certain days during the month of August, is given in the following table:

Date	1	2	3	4	6	7	8
Height	30.75	30.5	30.1	30.1	29.9	29.8	30.2

Represent these readings graphically.

(5) The amateur records for certain distances are given in the following table:

Distance in yards	100	120	150	200	250	300	440
Time in seconds	$9\frac{3}{5}$	$11\frac{4}{5}$	$14\frac{4}{5}$	$19\frac{4}{5}$	$24\frac{4}{5}$	$30\frac{3}{5}$	$47\frac{4}{5}$
	$\frac{48}{5}$	$\frac{59}{5}$	$\frac{74}{5}$	$\frac{99}{5}$	$\frac{124}{5}$	$\frac{153}{5}$	$\frac{239}{5}$

Represent these records graphically.

(6) R. Brown's salary is $3000 per year. His expenditures for the year are:

Rent	$450.00
Food	600.00
Clothing	300.00
Heating and Lighting	225.00
Recreation	150.00
Other expenses	600.00

The remainder of his salary he puts in the bank.

Construct a circular graph of this problem.

(7) The temperature on a winter day, taken every two hours for twenty-four hours, is shown in the following table:

Midnight	−4°	10 A.M.	15°	8 P.M.	10°
2 A.M.	−6°	12 Noon	22°	10 P.M.	2°
4 A.M.	−10°	2 P.M.	28°	Midnight	−3°
6 A.M.	−5°	4 P.M.	25°		
8 A.M.	5°	6 P.M.	20°		

Construct the temperature graph for the day. From your graph find at what times during the day the temperature was just zero.

(8) In an experiment to determine the effect of salt in water upon the boiling point, the following figures were obtained:

Percentage of salt in solution	0	3.2	6.3	9.4	12.2	17.9	22.9	27.7	31.9
Boiling Point in degrees C.	100	100.5	101.2	102	102.5	104.5	106	107.8	108.8

Plot a curve showing the above relation between boiling point and salt percentage.

Estimate:

(*a*) the boiling point if there is 20% of salt in solution.

(*b*) the percentage of salt required to produce a boiling point of 107°C.

(9) The following table gives the 'expectation of life' for men.

Age	1	10	20	30	40	50	60	70	80	90	100
Expectation of life	52	50	41	35	26	19	13	8	$4\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{2}$

Draw a graph to represent these figures.

From your graph find how many years on the average a student of your age may 'expect' to live.

If a man is 68 years old, how many more years of life may he expect?

(10) On the same diagram graph the metric equivalents of British measures, based on the following approximate relations:

100 pt.	=57 litres	100 in.	=254 cm.
100 lb.	=45.5 kg.	100 mi.	=161 km.
100 ac.	=40.5 hectares	100 yd.	=91.5 m.

Read from your diagram the metric equivalents of: 35 pt., 45 mi., 75 yd., 40 in., 60 lb., 48 ac.

Exercise XIXb

Note: Verify graphic solutions by calculation.

(1) Find graphically a fraction, with 30 as denominator, which is equal to $\frac{11}{25}$.

(2) 1 inch = 2.54 cm. (approximately). Find graphically the number of centimetres in 1.5 inches; in 4.75 inches.

(3) An examiner had marked a set of examination papers on the basis of 125 as full marks. He then changed the maximum to 100 and had therefore to alter the marks for each paper in the same ratio. Draw a graph by which the changed values could be read off. By means of this graph, find to the nearest mark, the new marks for papers on which the old marks were 48, 56, 75, 98, 110.

(4) A merchant marks his goods at an advance of 40% on the cost price. Construct a graph which would enable him to tell by inspection what should be the selling price of articles which cost $1, $2.50, $6.25, $10.

(5) A merchant gives a discount of 20% off the marked price of goods during a special sale. Draw a graph which would enable him to read off the reduced sale prices. Find from the graph the sale prices of articles previously marked at $1.50, $2.40, $4.75, $5.20, $10.

(6) A train travelling at the rate of 20 miles per hour passes a flag station at 11 a.m. A second train, travelling in the same direction at the rate of 30 miles per hour passes the flag station at 1 p.m. Show graphically when and where the second train will overtake the first.

(7) A steamer leaves port at the rate of 8 miles an hour at 3 a.m., Monday. At noon a dispatch boat starts in pursuit at the rate of 15 miles an hour. Show by

means of a graph when the dispatch boat will overtake the steamer. When will they be in sight of one another, supposing them to be visible 2 miles off?

(8) At 7 a.m., a train leaves Edmonton for Calgary, a distance of 200 miles, travelling at the rate of 30 miles an hour. At 8.30 a.m., a train leaves Calgary for Edmonton, travelling at the rate of 25 miles an hour. Show by means of a graph when and where they will meet.

(9) Show by means of a graph the amount of \$500, at 8% simple interest, at the end of 1, 2, 3, 4, 5 years. Is the amount proportional to the time?

(10) The distances travelled by a moving body were as follows:

Time from start (in seconds)	1	2	3	4	5
Distance (in feet)	2.4	4.7	7.3	9.6	12.05

The distances were measured approximately.

Show graphically that the distance travelled was proportional to the time.

How far did the body move in 8 seconds?

(11) The following is a time-table of a down train and an up train. A, B, C, D, E are railway stations:

Miles		Down	Up
0	A dep.	9.0 a.m.	arr. 11.20 a.m.
10	B dep.	9.20 a,m.	
25	C dep.	9.50 a.m.	
43	D dep.	10.26 a.m.	
50	E arr.	10.40 a.m.	dep. 9.30 a.m.

Draw a graph to represent the positions of the trains at different times. At what time, and how many miles from A, do the trains meet? At what times are the trains 12 miles apart?

(12) The electrical resistance of a wire of uniform cross section is directly proportional to the length. If the resistance of 100 yards is .25 ohms, construct a graph for reading off the resistance of any number of yards. What is the resistance of a half mile of this wire?

(13) Find graphically a fraction, with 30 as numerator, which is equal to $\frac{11}{18}$.

(14) A man has a journey of 80 miles to make. Complete the following table showing how long it will take at different rates of travelling.

Rate (miles per hour)	4	10	16	20	40
Time (hours)	20				

Draw a graph to show the relation between rate and time. How does the graph behave as the rate values become small? When the rate values become great? Use your graph to read off to the nearest minute the time taken (1) if he cycles at 9 miles an hour, (2) if he travels by car at 25 miles an hour, (3) if he travels by train at 30 miles an hour.

(15) The lengths of the shadows of different objects were measured at the same time.

A man 6 ft. in height cast a shadow of 7 ft. 4 in.
A pole 12 ft. high cast a shadow of 14 ft. 8 in.
A building 16 ft. high cast a shadow of 19 ft. 7 in.

From the above data find graphically (1) the height of a tree whose shadow is 30 ft., (2) the height of a flag-pole whose shadow is 45 ft.

(16) The formula $F=\frac{9}{5}C+32$ enables us to translate the reading on the Fahrenheit thermometer to the corresponding reading on the Centigrade thermometer. $F=$ the number of Fahrenheit units and $C=$ the number of Centigrade units.

Graph the above linear equation and determine from this graph the corresponding Fahrenheit readings for the following Centigrade readings: 5°, 10°, 20°, 25°, 30°.

Determine by the graph the Centigrade readings corresponding to the following Fahrenheit readings: 50°, 59°, 32°, 68°, 77°.

(17) When gas in a cylinder is put under pressure, the volume is reduced as the pressure is increased, the volume varying inversely as the pressure. If the volume of a gas is 16 c.c. when the pressure is 9 lb., find graphically the volume under the following pressures: 6 lb., 12 lb., 15 lb., 20 lb., 24 lb., 30 lb.

(18) A started to walk from P to Q, a distance of 12 miles, at 9.30 a.m. At 11 a.m. he met B, who had started to walk from Q to P at 10 a.m. If A arrives at Q at 12.30 p.m., when will B arrive at P? Solve graphically.

(19) In a 100 yards race A beats B by 20 yards, and B beats C by 10 yards. How many yards start can A give C in 100 yards so that they may run a dead heat? Solve graphically.

(20) A retailer makes a profit of 50% when he sells an article to a consumer for $60. What did the article cost

him? If the wholesaler made a profit of 25% when he sold the article to the retailer, what did the article cost the wholesaler? Solve graphically.

(21) A merchant marks his goods at $33\frac{1}{3}\%$ above the cost price, but allows a discount of 10% for cash. What is the cost price of an article for which he receives $36 cash? Solve graphically.

(22) A train leaves P at 10.15 a.m. for Q, which is 90 miles distant, and travels at the rate of 30 miles per hour. At 10.42 a.m., a train which travels at the rate of 45 miles an hour, leaves Q for P. Find graphically when and where the trains will meet.

(23) The formula $S=16t^2$ represents the relation between the distance a body falls and the time it is falling. S represents distance in feet, and t represents time in seconds. Draw the graph of this formula and from your graph answer the following questions: (1) How far will a body fall in $3\frac{1}{2}$ seconds? in $5\frac{1}{2}$ seconds? (2) How long will it take a body to fall 40 feet? 130 feet? 300 feet?

CHAPTER XX

LOGARITHMIC CALCULATION

A. Fundamental Conceptions

The Logarithm of a number to any base is the index of the power to which the base must be raised to produce the number.

2^0	=	1
2^1	=	2
2^2	=	4
2^3	=	8
2^4	=	16
2^5	=	32
2^6	=	64
2^7	=	128
2^8	=	256
2^9	=	512
2^{10}	=	1024
2^{11}	=	2048
2^{12}	=	4096
2^{13}	=	8192
	etc.	

In the table shown 2 may be called the base; 0, 1, 2, 3, 4, etc., which are indices of the powers of the base 2, are the logarithms of the numbers 1, 2, 4, 8, 16, etc., to the base 2. 0 = the logarithm of 1 to the base 2, 2 is the logarithm of 4 to the base 2, 7 is the logarithm of 128 to the base 2, etc. The logarithm of 128 to the base 2 may be more briefly expressed thus, $\log_2 128$. $128 = 2^7$, and $\log_2 128 = 7$, are equivalent statements.

By reference to this table it may be shown that the product of numbers may be obtained by the addition of indices, the quotient in a division by the subtraction of indices, a power of a number by multiplying an index, and a root of a number by dividing an index (in all cases by the aid of the table).

I. Multiplication

To find the product of 16 and 512.

From the table it may be seen that $16 = 2^4$, and $512 = 2^9$.

$16 \times 512 = 2^4 \times 2^9 = 2^{4+9} = 2^{13}$, which the same table shows to be 8192.

Then $16 \times 512 = 8192$.

The product has been found by the addition of indices and reference to the tables. This illustrates logarithmic calculation as applied to multiplication.

Referring to this example it may be observed that $\log_2 16 = 4$, $\log_2 512 = 9$, and $\log_2 8192$, their product, is $4+9$. The logarithm of the product of the two factors is the sum of the logarithms of the factors.

Application: To multiply 64 by 32 by logarithms.

$\text{Log}_2 64 = 6$, $\log_2 32 = 5$

Then $\log_2(64 \times 32) = 6+5 = 11 = \log_2 2048$

Then $64 \times 32 = 2048$.

II. DIVISION

To divide 4096 by 256.

$4096 = 2^{12}$, $256 = 2^8$

$$\frac{4096}{256} = \frac{2^{12}}{2^8} = 2^{12-8} = 2^4 = 16$$

Otherwise: Since $\log_2 4096 = 12$
and $\log_2 256 = 8$

$$\text{and } \log_2 \frac{4096}{256} = 12-8, \left(\text{since } \frac{4096}{256} = 2^{12-8}\right)$$

$$\text{then } \log_2 \frac{4096}{256} = \log_2 4096 - \log_2 256.$$

The logarithm of the quotient is the logarithm of the dividend diminished by the logarithm of the divisor.

And since $4 = \log_2 16$, $\dfrac{4096}{256} = 16$.

Application: To divide 2048 by 64 by logarithms.

$$\log_2 2048 = 11$$

$$\log_2 64 \quad = \ 6$$

$$\log_2 \frac{2048}{64} = \log_2 2048 - \log_2 64 = 11 - 6 = 5$$

But $5 = \log_2 32$

Then $\dfrac{2048}{64} = 32.$

III. Involution

To find the second power of 64.

$64 = 2^6$

$(64)^2 = (2^6)^2 = 2^{2\times 6} = 2^{12} = 4096$

$64^2 = 4096.$

Otherwise:

$\log_2 64 \quad = 6$

$\log_2 (64)^2 = 2\times 6$, (since $64^2 = 2^{2\times 6}$)

$\log_2 (64)^2 \ = 2\times \log_2 64, \ = 12$

That is, the log of the square of a number is the log of the number multiplied by 2, the index of the power to which the number is raised. Since $12 = \log_2 4096$, $(64)^2 = 4096$.

Application: To raise 16 to the third power.

$\log_2 16 = 4$

$\log_2 (16)^3 = 4\times 3 = 12 = \log 4096$

$\therefore \ 16^3 = 4096.$

IV. Evolution

To find the cube root of 512.

$512 = 2^9$

$\sqrt[3]{512} = (512)^{\frac{1}{3}} = (2^9)^{\frac{1}{3}} = 2^{\frac{9}{3}} = 2^3 = 8.$

Otherwise:

$\log_2 512 = 9$

$\log_2 \sqrt[3]{512} = \frac{9}{3}$, (since $\sqrt[3]{512} = 2^{\frac{9}{3}}$)

$\log_2 \sqrt[3]{512} = \frac{1}{3}\log_2 512.$

That is the log of the cube root of a number is the logarithm of the number, divided by 3, the index of the root to be taken.

$\log \sqrt[3]{512} = \frac{1}{3}$ of $9 = 3$

$3 = \log_2 8$

$\sqrt[3]{512} = 8.$

Application: To find the 5th root of 1024.

$\log_2 1024 \quad = 10$

$\log_2 \sqrt[5]{1024} = \frac{10}{5} = 2 = \log_2 4$

$\therefore \sqrt[5]{1024} = 4.$

In these examples the index laws as established in Algebra are assumed. If it be possible to express not only 1, 2, 4, 8, 16, etc., but all numbers, as powers of 2, which of course will involve fractional indices, any multiplication or division, raising to any power or taking any root, may be accomplished by these methods.

The principles just illustrated may be proved generally as follows:

Let the base be a.

(1) $\text{Log}_a\text{MN} = \log_a\text{M} + \log_a\text{N}$

Let $\text{M} = a^x$, that is $\log_a\text{M} = x$

and $\text{N} = a^y$, that is $\log_a\text{N} = y$

$\text{MN} = a^x.a^y = a^{x+y}$

Then $\log_a\text{MN} = x + y = \log_a\text{M} + \log_a\text{N}$.

(2) $\text{Log}_a\frac{\text{M}}{\text{N}} = \log_a\text{M} - \log_a\text{N}$

Let $\text{M} = a^x$, that is $\log_a\text{M} = x$

and $\text{N} = a^y$, that is $\log_a\text{N} = y$

$\frac{\text{M}}{\text{N}} = \frac{a^x}{a^y} = a^{x-y}$

Then $\log_a\frac{\text{M}}{\text{N}} = x - y = \log_a\text{M} - \log_a\text{N}$.

(3) $\text{Log}_a\text{M}^r = r\log_a\text{M}$

Let $\text{M} = a^x$, that is $\log_a\text{M} = x$

$\text{M}^r = (a^x)^r = a^{xr}$

Then $\log_a\text{M}^r = xr = r\log_a\text{M}$.

(4) $\text{Log}_a\sqrt[r]{\text{M}} = \frac{1}{r}\log_a\text{M}$

Let $\text{M} = a^x$, that is $\log_a\text{M} = x$

$\sqrt[r]{\text{M}} = \text{M}^{\frac{1}{r}} = (\text{a}^x)^{\frac{1}{r}} = \text{a}^{\frac{x}{r}}$

Then $\log_a\sqrt[r]{\text{M}} = \frac{x}{r} = \frac{1}{r}(x) = \frac{1}{r}\log_a\text{M}$.

B. Common Logarithms

Except for purposes of illustration 2 is not used as a base for logarithms. Originally logarithms were calcu-

lated to a base approximately 2.7182818. For important reasons having to do with our system of writing numbers logarithms calculated to the base 10 are more convenient than logarithms calculated to the base 2.7182818, and the logarithms obtained to this latter base (Naperian logarithms, so called from their discoverer) have been changed to logarithms to the base 10. Logarithms to the base 10 are called common logarithms.

$\text{Log}_{10}2$ may be written "log 2" as the base is understood to be 10 if not written.

$$\begin{aligned}
\text{Log } 1 &= 0, \text{ since } 1 = 10^0 \\
\text{Log } 10 &= 1, \text{ since } 10 = 10^1 \\
\text{Log } 100 &= 2, \text{ since } 100 = 10^2 \\
\text{Log } 1000 &= 3, \text{ since } 1000 = 10^3, \text{ etc.}
\end{aligned}$$

The logarithm of a number between 1 and 10 lies between 0 and 1, that is, it is a positive proper fraction, or decimal. The logarithm of a number between 10 and 100 lies between 1 and 2, or it is 1+ a decimal. The logarithm of a number between 100 and 1000 lies between 2 and 3, or it is 2+ a decimal.

$$\begin{aligned}
\text{For example, } \log 8 &= 0.90309 \\
\log 64 &= 1.80618 \\
\log 200 &= 2.30103, \text{ etc.}
\end{aligned}$$

The whole number in a logarithm is called the *characteristic* and the decimal is called the *mantissa* (pl. mantissae). It is apparent then that the characteristic of the logarithm of a number of 1 digit, or with 1 digit in its integral part, is 0. If there are 2 digits in the integral part the characteristic is 1, 3 digits give a characteristic 2,

and in general, the characteristic is one less than the number of digits in the integral part of the number.

The logarithms of numbers less than unity are negative.

$$\text{Log } .1 = -1, \text{ since } .1 = \frac{1}{10} = 10^{-1}$$

$$\text{Log } .01 = -2, \text{ since } .01 = \frac{1}{100} = 10^{-2}$$

$$\text{Log } .001 = -3, \text{ since } .001 = \frac{1}{100} = 10^{-3}$$

$$\text{Log } .0001 = -4, \text{ since } .0001 = \frac{1}{10000} = 10^{-4}, \text{ etc.}$$

The logarithms of fractions between 0 and .1, that is, those which have no zero immediately following the decimal point when expressed as decimals, lie between 0 and -1, that is, they are negative decimals. For example log $.2 = -.69897$. For convenience in tabulating and using these logarithms the mantissae are always positive in logarithmic tables. $-.69897$ is equivalent to $-1 + .30103$; then log $.2 = -1 + .30103$, written $\bar{1}.30103$. The logarithms of fractions between .1 and .01, that is, fractions which as decimals have one zero between the decimal point and the first significant digit, lie between -1 and -2, and are written $-2+$ a decimal. Log $.03 = \bar{2}.47712$.

Logarithms of decimal fractions with 2 zeros between the decimal point and the first significant digit are $-3+$ a decimal. Log $.005 = \bar{3}.69897$.

Thus the characteristic of the logarithm of a decimal is negative and one *more* than the number of zeros between

the decimal point and the first significant digit. In tabulating logarithms, whether of numbers greater than unity or less than unity, the characteristics are omitted since they may be written by rule. This is an advantage the base 10 has over the base 2.7182818.

In logarithmic calculation involving negative characteristics the fact that the mantissa is always positive must not be forgotten in the operations involved.

Examples:

(1) Given log $16 = 1.20412$ and log $.35 = \bar{1}.54407$, to find the log of their product.

By addition, log $(16 \times .35) = 0.74819$.

(2) Given log 16 and log .35, to find the log of $16 \div .35$.

$$
\begin{array}{ll}
\text{Log } 16 & = 1.20412 \\
\text{Log } .35 & = \bar{1}.54407 \\
\hline
\text{Log } (16 \div .35) & = 1.66005, \text{ the difference.}
\end{array}
$$

To subtract -1, add $+1$, as in Algebra.

Note: If the logarithm to be subtracted is the larger, the characteristic in the difference will be negative.

(3) Given log $.35 = \bar{1}.54407$, to find the log of $(.35)^4$.

This will be $\bar{1}.54407$ multiplied by 4

$$
\begin{array}{r}
\bar{1}.54407 \\
4 \\
\hline
\bar{2}.17628
\end{array}
$$

The result is equivalent to $-4 + 2.17628$.

(4) Given log $.35 = \bar{1}.54407$, to find the log of the cube root of .35.

It is necessary to divide 1.54407 by 3.

$\bar{1}.54407$ may be written $-3+2.54407$. Since the divisor is 3, make the negative part the next multiple of 3 and correct by adding a positive number equal to the increase in the negative characteristic.

$$-3+2.54407 \div 3 = -1+84802 = \bar{1}.84802.$$

C. Tables of Logarithms

Logarithms calculated to four places of decimals are used chiefly for illustration. Five place tables are suitable for many purposes and for very accurate work seven place tables are available.

Given $\log 8168526 = 6.9121437$

$$\text{Log } 816852.6 = \log \frac{8168526}{10} = \log 8168526 - \log 10$$

$$= 6.9121437 - 1 = 5.9121437$$

Similarly $\log 81685.26 = 6.9121437 - 2 = 4.9121437$
and $\log 816852600 = 6.9121437 + 2 = 8.9121437$

$$\log .8168526 \quad = 6.9121437 - 7 = \bar{1}.9121437$$
$$\log .0008168526 = 6.9121437 - 10 = \bar{4}.9121437.$$

From this it appears that *the mantissa of the logarithm of a number composed of any sequence of significant digits, remains the same no matter where the decimal point may be placed.* In looking up the logarithm of a number the decimal point may be disregarded as far as the mantissa is concerned as it affects only the characteristic. This is another respect in which the adoption of the base 10 has simplified the tabulation of logarithms.

To find the logarithm of 614.32. 614 may be found on the "600 page" (i.e. the page beginning with the logarithm of 600). To the right of 614, in the 4th column under the heading 3 will be found log $6143 = 78838$. There is still the 2 to consider. Add to 78838, 2 tenths of the difference between this log and the next one, viz: 78845. $\frac{2}{10}$ of $7 = \frac{14}{10} = 1$ to the nearest unit.

Put in characteristic and decimal points.

Then log $614.32 = 2.78839$

Note: Since log $61430 = .78838$

and log $61440 = .78845$

omitting decimal points and characteristics, log of 61432 will include approximately $\frac{2}{10}$ of the difference.

To find the number of which $\overline{2}.46618$ is the logarithm.

Logs beginning with 46 will be found on the "250 page." Opposite 292 will be found 465. Tracing to the right, under the column headed 5 will be found 46613. Under 6 will be found 46627.

Then log $29250 = 46613$

and log $29260 = 46627$ (omitting decimal points).

The difference of 14 in the logs comes from a difference of 10 in the numbers. The given log is 46618. This is 5 more than 46613

14 comes from difference 10

5 comes from difference $\frac{10 \times 5}{14} = 4$.

Then .46618 is the mantissa of the log of 29254.

Then $\overline{2}.46612 = \log .029254$.

Examples:

(1) Find the edge of a cube containing 13824 cubic feet.

The edge is $\sqrt[3]{13824}$

$$\text{Log } 13824 = 4.14063$$

$$\text{Log } \sqrt[3]{13824} = \frac{4.14063}{3} = 1.38021$$

$$1.38021 = \log 24$$

Then the edge is 24 feet.

(2) Find the amount of \$30 in 11 years allowing interest at 3% per annum, added half-yearly.

$$\text{Amount} = 30(1.015)^{22}$$

$$\begin{aligned} \text{Log } 30(1.015)^{22} &= \log 30 + \log (1.015)^{22} \\ &= \log 30 + 22(\log 1.015) \\ &= 1.47712 + 22(.00647) \\ &= 1.47712 \\ &+ .14234 \\ \hline &= 1.61946 \\ &= \log 41.635 \end{aligned}$$

Amount = \$41.64.

(3) In how many years will \$1 amount to \$4 at 5% per annum, interest added yearly.

Let the number of years be x

$$\begin{aligned} \text{Then } (1.05)^x &= 4 \\ \log (1.05)^x &= \log 4 \\ x \log 1.05 &= \log 4 \\ x &= \frac{\log 4}{\log 1.05} = \frac{.60206}{.02119} = 28.4, \text{ a little over} \end{aligned}$$

28 years.

Exercise XXa

(1) Find from the tables the logarithms of:

(*a*) 2, 6, 20, 60, 200, 600, 2000, 6000, 10^5.

(*b*) 1234, 229.8, 34.39, .4808, .005746.

(*c*) 678550, 72.934, .84472, .0047628.

(2) Of what numbers are the following the logarithms:

(*a*) .95405, .47712, 2.25334, $\bar{1}.11227$.

(*b*) 4, 3.89254, $\bar{2}.59956$, $\bar{1}.01452$.

(*c*) 1.25345, 2.60165, $\bar{5}.69944$, .94970.

(3) Use logarithmic calculation in the following: (4 significant digits in answer).

(*a*) 3.563×2.412; 5.624×2.5; 3.007×1.862.

(*b*) 6297×43.65; $(82.64)^2$; $(2.834)^2$.

(*c*) $9.472\div 2.864$; $6843\div .2382$.

(*d*) $48.79\div 95.42$; $\sqrt{652.4}$; $(8.475)^{\frac{1}{2}}$

(4) By means of logarithms find the value of the following: (5 significant digits).

(*a*) $2.23\times 1.76\times 3.54\times 4.67$.

(*b*) $(1.362)^3$; $(3.895)^4$; $\sqrt[3]{1523}$; $\sqrt[4]{.0016}$.

(*c*) $.04782\times 6.512\times 97.68$.

(5) Obtain the results of the following indicated operations:

(*a*) $\dfrac{768\times 495}{237\times 399}$

(*b*) $\dfrac{(7.96)^2}{(42.31)}$

(*c*) $\sqrt[3]{\dfrac{24.96\times 8.35}{12.41\times 7.24}}$

Exercise XXb

By the use of logarithms solve the following problems:

(In these problems use the approximation 3.1416 for π.)

(1) Find the number of acres in a rectangular field 457 yd. long and 288 yd. wide.

(2) Find the number of cubic feet of air space in a room whose length is 36.4 ft., width 28.6 ft., and height 11.8 ft.

(3) The area of a rectangular field is 3.75 acres and the length is 156 yd. Find the width.

(4) The volume of a cylinder is given by the formula $V = \pi r^2 h$ where r = radius of base, h = height. Find V when:

(*a*) $h = 9.4$ in., $r = 3.2$ in.

(*b*) $h = 24.6$ cm., $r = 12.5$ cm.

(*c*) $h = 32.8$ in., $r = 8.4$ in.

(5) Find the height of a cylinder whose volume is 963 cu. in., and the radius of the base 5.8 in.

(6) The volume of a sphere is given by $\frac{4}{3}\pi r^3$ where r = radius.

(*a*) Find the volume of a sphere whose radius is 7.6 in.

(*b*) Find the radius of a sphere whose volume is (1) 1000 c.c., (2) 15 cu. ft., (3) 12.5 cu. in.

(7) The volume of the material in a hollow cylinder is given by the formula $V = \pi h(R+r)(R-r)$ where R = external radius, r = internal radius, h = height. Find:

(*a*) The number of cu. in. of iron in a hollow cylinder where $R = 6.2$ in., $r = 4.9$ in., and $h = 14.6$ in.

(*b*) The weight of iron in a hollow cylinder where R = 4.3 in., r = 2.7 in., and h = 9.4 in., the specific gravity of iron being 7.7.

(8) Given that 1 in. = 2.54 cm., find (1) the number of sq. cm., in 1 sq. in., (2) the number of cubic centimetres in 1 cu. in.

(9) Given that 1 lb. = 453.59 grams. find the number of pounds in a kilogram.

(10) Find the weight in tons of a block of stone 8.5 ft. long, 5.4 ft. wide, and 3.8 ft. thick, the specific gravity of the stone being 2.65. 14.444

(11) Find the area of a circle of radius 8.74 cm.

(12) Find, in yards, the radius of a circular pond whose area is 2 acres.

(13) The circumference of the earth is approximately 40,000,000 metres. How many miles is this? (1 m. = 39.37 in.)

(14) A 12-inch cube is melted down and made into a sphere. Find its radius.

(15) Given 1 metre = 39.37 inches, convert:

(1) 635 m. into yards.
(2) 1 yard into metres.
(3) 45.8 m. into inches.
(4) 1 km. into miles.
(5) 1 in. into centimetres.
(6) 5.4 in. into millimetres.

Exercise XXc

(1) Sound travels in still air at 32°F. at the rate of 1090 ft. per second. Sound travels in water about 4.25 as fast as in air. How many seconds earlier would the

sound of a torpedo exploded under water $1\frac{1}{2}$ miles away reach you by water than by air, the temperature of the air being at freezing point?

(2) Find the amount of \$1200 in 10 years, the rate of interest being 5% compounded annually.

(3) The sides of a triangular field are 328 yd., 462 yd., and 546 yd. Find, in acres, the area of the field.

(4) Find the present value of \$4000 due in 5 years, money being worth 6% per annum.

(5) A owes B \$200 due in 2 years; \$250 due in 3 years; \$300 due in 4 years. What sum paid now would cancel the debt, money being worth 5% per annum, compounded yearly?

(6) At 5% per annum, compounded quarterly, \$1200 amounts to \$1972.23 in a certain time. Find the time.

(7) A merchant estimates that at the end of every month the value of his unsold goods is only 99% of their value at the beginning of the month. He calculates in this way that the value of part of his stock is only half what he gave for it. How long has he had the goods in his possession.

(8) The population of three municipalities are 10,941, 12,311, and 25,992. If \$26,840 is to be distributed among them in proportion to the population, how much money will each receive?

(9) The distance of the star Castor from the earth is approximately 9.59×10^{13} miles. The velocity of light is 186,330 miles per second. Find the number of years taken by light from Castor to reach us.

(10) In a certain airship the lifting force of 100 cu. ft. of gas is 8.45 pounds. Find the lifting force after 20

hours, of an airship which contained 354,276 cu. ft. of gas originally, assuming that it loses 1% of its volume of gas in 24 hours.

(11) A man motors 985.8 miles in 5.5 days, travelling 10.4 hours a day. At the same average rate how many hours a day must he travel to cover 3967 miles in 24.8 days?

(12) The time in seconds of a complete oscillation of a pendulum of length l feet is given by the formula $t = 2\pi\sqrt{\frac{l}{g}}$ where $g = 32$. Find the time of a complete oscillation of a pendulum 1 metre long. (1 m. = 39.37 in.)

(13) The mass of the earth is 6×10^{21} tons approximately. The mass of the earth is to that of the moon as 81.5 : 1. Find the mass of the moon.

(14) When a boy was 16 years old $7500 was deposited to his credit in a bank which paid interest at the rate of 4% per annum, compounded half-yearly. This sum, with interest, was allowed to accumulate till the boy was 21 years old. How much was there then to his credit in the bank?

(15) A vacuum pump was constructed which at each stroke would remove one-eighth of the air in the receiver at the beginning of the stroke. After how many strokes would the amount of air left be 1/1000 of the air at the beginning of the operation?

(16) Three municipalities unite to maintain a High School and agree to share the cost in proportion to the attendance from the respective municipalities. The total days attendance were 4152 from the first, 9650 from the second and 12147 from the third. Divide the cost of maintenance, $12540, among them.

WEIGHTS AND MEASURES

Canadian Money

10 mills = 1 cent, ct.

100 cents = 1 dollar, $.

The mill is a theoretical unit used for expressing rates of taxation but not recognized in commercial transactions.

The following Canadian coins are authorized by the Currency Act:

Gold	: $20, $10, $5, $2½.
Silver	: $1, 50 cents, 25 cents, 10 cents, 5 cents.
Nickel	: 5 cents.
Bronze	: 1 cent.

Gold coins have been issued in denominations of $10 and $5 only. The first Canadian gold coin was issued by the mint, 10th May, 1912.

The Government issues paper notes of denominations 25 cents, $1, $2, $5, $500 and $1000, all legal tender.

United States Money

The United States gold dollar and the Canadian gold dollar are equal in value. The United States 10 cent coin is called a dime and the $10 gold piece an eagle, but in business transactions sums of money are expressed in dollars and cents.

Note: The Mexican Monetary unit is called a dollar and is worth 49.8 cents. The dollar of the Straits Settlements is worth 56.7 cents.

British Money

The Pound Sterling is the monetary unit in Great Britain, Ireland, Australia, New Zealand, South Africa and many other parts of the Empire.

The pound is equivalent to \$4.8665, or \$4.86$\frac{2}{3}$ approximately.

4 farthings (f. or $\frac{1}{4}$d.)	= 1 penny, d.
12 pence	= 1 shilling, s.
20 shillings	= 1 pound, £.
21 shillings	= 1 guinea.
5 shillings	= 1 crown.
2 shillings	= 1 florin.

The gold coin worth a pound is called a sovereign. The guinea is no longer coined.

Avoirdupois Weight

The standard weight in Canada is the Imperial pound, the weight of a certain piece of platinum kept in the Exchequer Office, London.

16 ounces (oz.)	= 1 pound, lb.
100 pounds	= 1 hundredweight, cwt.
2000 pounds	= 1 ton, T.
1 pound	= 7000 grains, gr.

In Great Britain the hundredweight has been 112 lbs. since the days of Edward III. A fourth part of this, 28 lbs., is called a quarter, and an eighth part, 14 lbs., a stone. The British ton is 2240 lbs., which has certain restricted uses in this country, chiefly in mining and calculating customs duties. In Canada 2240 lbs. is known as the long ton, in contrast to the 2000 lb. ton, or short ton.

Avoirdupois weight (or metric weights) must be used in buying or selling all articles (including drugs) by weight in Canada except in the case of gold, silver, platinum or precious stones, when Troy weight may be used.

Troy Weight

24 grains (gr.)	=1 pennyweight, dwt.
20 pennyweights	=1 ounce, oz.
12 ounces	=1 pound, lb.

Apothecaries' Weight

These weights are no longer used in buying and selling drugs, but are still used in compounding medicines.

20 grains	=1 scruple,	℈ or sc.
3 scruples	=1 dram,	ʒ or dr.
8 drams	=1 ounce,	℥ or oz.
12 ounces	=1 lb.,	lb.

Capacity

A fluid ounce of water weighs 1 oz. Avoirdupois and contains 8 fluid drachms (=drams). A fluid drachm contains 60 minims or drops.

2 pints (pt.)	=1 quart	qt.
4 quarts	=1 gallon,	gal.

An Imperial gallon of water at 62°F. weighs 10 lbs. and contains 277.274 cubic inches. The United States gallon contains 231 cubic inches and is thus about $\frac{5}{6}$ of the Imperial gallon.

A cubic foot of water actually weighs 62.321 lbs. at 62°F. (62.42 lbs. at maximum density) but for ordinary

purposes the weight is taken as $62\frac{1}{2}$ lbs. or 1000 oz. Thus a cubic foot contains approximately $6\frac{1}{4}$ gallons or 25 quarts.

Dry Measure

4 quarts	= 1 gallon,	gal.
2 gallons	= 1 peck,	pk.
4 pecks = 8 gallons	= 1 bushel,	bush.

A Dominion bushel contains 2218.19 cubic inches. The bushel measure is seldom used in buying or selling. Most dry commodities that were formerly sold by volume, and measured in bushels, are now sold by weight. Fruit may be sold in bushel baskets.

There is no legal number of pounds per bushel for grains but the following weights are generally recognized:

Blue Grass Seed	14 lbs.	Indian Corn	56 lbs.
Oats	34 "	Rye	56 "
Buckwheat	48 "	Wheat	60 "
Timothy Seed	48 "	Peas	60 "
Barley	48 "	Clover Seed	60 "
Flax Seed	54 "		

Under the Root Vegetables Act, (1922) potatoes, turnips, carrots, parsnips, onions, beets and other root vegetables must be sold by weight and not by the bushel.

A quarter (British) of wheat is 480 lbs.

An apple barrel contains 7056 cubic inches. When pears or quinces are sold by the barrel the barrel must be the same size as the apple barrel. An apple box, or crate, contains 2174 cubic inches. A pear or crabapple box contains 1760 cubic inches.

Linear Measure

The standard of length is the yard.

12 inches (in. or ″)	= 1 foot, (ft. or ′)	
3 feet	= 1 yard, yd.	
$5\frac{1}{2}$ yards		
= $16\frac{1}{2}$ feet	= 1 rod, rd.	
320 rods		
= 1760 yards		
= 5280 feet		
= 63360 inches	= 1 mile, mi.	

A surveyor's chain is 4 rods (66 ft.) long and contains 100 equal links.

4 inches	= 1 hand.	Used in measuring horses.
1 knot	= 6080 feet	Used at sea.
6 feet	= 1 fathom	Used at sea.
120 fathoms	= 1 cable length	Used at sea.
3 knots	= 1 league	Used at sea.

Square Measure

144 square inches (sq. in.)	= 1 square foot, sq. ft.	
9 square feet	= 1 square yard, sq. yd.	
$30\frac{1}{4}$ square yards	= 1 square rod, sq. rd.	
160 square rods		
= 4840 square yards	= 1 acre, A.	
640 acres	= 1 square mile.	
10 square chains	= 1 acre.	
100 square feet	= 1 square of roofing or flooring.	
640 acres	= 1 section	Used in the prairie provinces.
36 sections	= 1 township	Used in the prairie provinces.

Cubic Measure

1728	cubic inches	= 1 cubic foot, cu. ft.
27	cubic feet	= 1 cubic yard, cu. yd.
128	cubic feet	= 1 cord, cd.
$24\frac{3}{4}$	cubic feet	= 1 perch of stone.

Time

60 seconds	= 1 minute, min.
60 minutes	= 1 hour, hr.
24 hours	= 1 day, da.
365 days	= 1 common year, yr.
366 days	= 1 leap year.
12 calendar months	= 1 year.

The solar year contains approximately 365 da. 5 hr. 48 min. 46 sec.

The proper fixing of leap years should make this the average year. The Gregorian calendar which is now in use in almost all countries of the world does this almost exactly. Every fourth year is a leap year except when the number of the year is divisible by 100, in which case it is a leap year if divisible by 400 but not otherwise. The year 1900 was not a leap year but the year 2000 will be a leap year.

February, has 29 days in leap year and 28 days in other years. April, June, September and November have 30 days each. Each of the other months has 31 days.

Angular Measure

60 seconds ($''$)	= 1 minute, $'$.
60 minutes	= 1 degree, $^\circ$.
90 degrees	= 1 right angle, rt. $\llcorner$.
360 degrees	= 1 circle.

Other Tables

12 things	=1 dozen.
12 dozen	=1 gross.
12 gross	=1 great gross.
20 things	=1 score.
24 sheets of paper	=1 quire.
20 quires	=1 ream.

Metric Units

The fundamental unit of length is the metre, 39.37079 inches. The units of area, volume, capacity and weight are all derived from the metre.

Throughout, the prefixes, deci, centi, milli, micro, denote values 1/10, 1/100, 1/1000 and 1/1,000,000 respectively, of the unit to which they are prefixed. The prefixes, deca, hecto, kilo, myria, mega, denote values 10, 100, 1000, 10,000 and 1,000,000 times the unit to which they are prefixed.

1 metre (m)	=	10 decimetres,	dm.
	=	100 centimetres,	cm.
	=	1,000 millimetres,	mm.
	=	1,000,000 micrometres,	μm.
10 metres	=	1 decametre,	Dm.
100 metres	=	1 hectometre,	Hm.
1,000 metres	=	1 kilometre,	Km.
10,000 metres	=	1 myriametre,	Mm.
1,000,000 metres	=	1 megametre,	Mgm.

Otherwise

1,000 micrometres	=1 millimetre
10 millimetres	=1 centimetre.
10 centimetres	=1 decimetre.
10 decimetres	=1 metre.
10 metres	=1 decametre.
10 decametres	=1 hectometre.
10 hectometres	=1 kilometre.
10 kilometres	=1 myriametre.
100 myriametres	=1 megametre.

Area

100 square millimetres	=1 square centimetre, sq. cm.
100 square centimetres	=1 square decimetre, sq. dm
100 square decimetres	=1 square metre, sq. m.
100 square metres	=1 square decametre, sq. Dm.
100 square decametres	=1 square hectometre, sq. Hm.
100 square hectometres	=1 square kilometre, sq. Km.

Land Measurement

1 square decametre is called an are

1 square metre is called a centare (or centiare)

1 square hectometre is called a hectare

Hence:—100 centares = 1 are

100 ares = 1 hectare.

The hectare is 2.471 acres.

Capacity or Volume

1000 cubic millimetres	=1 cubic centimetre, c.c.
1000 cubic centimetres	=1 cubic decimetre
1000 cubic decimetres	=1 cubic metre, etc.

The cubic metre is called a stere, which is the unit of volume corresponding to the English cubic yard.

1 stere = 1.308 cubic yards = 35.317 cubic feet.

The cubic decimetre is the unit of capacity and is called a litre (*l.*)

1 litre = 10 decilitres, dl.
= 100 centilitres, cl.
= 1000 millilitres, ml.

10 litres = 1 decalitre, Dl.
100 litres = 1 hectolitre, Hl.
1000 litres = 1 kilolitre, Kl.

A hectolitre is 2.7512 (or approximately $2\frac{3}{4}$) bushels.

The weight of 1 cubic centimetre of distilled water at 0° Centigrade is the unit of weight. It is called the gramme.

1 gramme (or gram)	= 10 decigrams, dg.
	= 100 centigrams, cg.
	= 1000 milligrams, mg.
10 grams	= 1 decagram, Dg.
100 grams	= 1 hectogram, Hg.
1000 grams	= 1 kilogram, Kg.
1000 kilograms	= 1 Tonneau, or metric ton. T.

A litre of water weighs a kilogram.
1 gram = 15.432 grains.
1 kilogram = 2.2046 lbs.
1 Tonneau = 2204.6 lbs.

SIMPLE INTEREST TABLES—6%

Principal	15 days	18 days	30 days	31 days	32 days	33 days	34 days
1000	2.466	2.959	4.932	5.096	5.260	5.425	5.589
1100	2.712	3.255	5.425	5.605	5.786	5.967	6.148
1200	2.959	3.551	5.918	6.115	6.312	6.510	6.707
1300	3.205	3.847	6.411	6.625	6.838	7.052	7.266
1400	3.452	4.142	6.904	7.134	7.364	7.595	7.825
1500	3.699	4.438	7.397	7.644	7.890	8.137	8.384
1600	3.945	4.734	7.890	8.153	8.416	8.679	8.942
1700	4.192	5.030	8.384	8.663	8.942	9.222	9.501
1800	4.438	5.326	8.877	9.173	9.468	9.764	10.060
1900	4.685	5.622	9.370	9.682	9.995	10.307	10.619
2000	4.932	5.918	9.863	10.192	10.521	10.849	11.178
2100	5.178	6.214	10.356	10.701	11.047	11.392	11.737
2200	5.425	6.510	10.849	11.211	11.573	11.934	12.296
2300	5.671	6.805	11.342	11.721	12.099	12.477	12.855
2400	5.918	7.101	11.836	12.230	12.625	13.019	12.414
2500	6.164	7.397	12.329	12.740	13.151	13.562	13.973
2600	6.411	7.693	12.822	13.249	13.677	14.104	14.532
2700	6.658	7.989	13.315	13.759	14.203	14.647	15.090
2800	6.904	8.285	13.808	14.268	14.729	15.189	15.649
2900	7.151	8.581	14.301	14.778	15.255	15.732	16.208
3000	7.397	8.877	14.795	15.288	15.781	16.274	16.767
3100	7.644	9.173	15.288	15.797	16.307	16.816	17.326
3200	7.890	9.468	15.781	16.307	16.833	17.359	17.885
3300	8.137	9.764	16.274	16.816	17.359	17.901	18.444
3400	8.384	10.060	16.767	17.326	17.885	18.444	19.003
3500	8.630	10.356	17.260	17.836	18.411	18.986	19.562
3600	8.877	10.652	17.753	18.345	18.937	19.529	20.121
3700	9.123	10.948	18.247	18.855	19.463	20.071	20.679
3800	9.370	11.244	18.740	19.364	19.989	20.614	21.238
3900	9.616	11.540	19.233	19.874	20.515	21.156	21.797
4000	9.863	11.836	19.726	20.384	21.041	21.699	22.356
4100	10.110	12.132	20.219	20.893	21.567	22.241	22.915
4200	10.356	12.427	20.712	21.403	22.093	22.784	23.474
4300	10.603	12.723	21.205	21.912	22.619	23.326	24.033
4400	10.849	13.019	21.699	22.422	23.145	23.868	24.592
4500	11.096	13.315	22.192	22.932	23.671	24.411	25.151
4600	11.342	13.611	22.685	23.441	24.197	24.953	25.710
4700	11.589	13.907	23.178	23.951	24.723	25.496	26.268
4800	11.836	14.203	23.671	24.460	25.249	26.038	26.827
4900	12.082	14.499	24.164	24.970	25.775	26.581	27.386
5000	12.329	14.795	24.658	25.479	26.301	27.123	27.945
5100	12.275	15.090	25.151	25.989	26.827	27.666	28.504
5200	12.822	15.386	25.644	26.499	27.353	28.208	29.063
5300	13.068	15.682	26.137	27.008	27.879	28.751	29.622
5400	13.315	15.978	26.630	27.518	28.405	29.293	30.181

SIMPLE INTEREST TABLES—6%

15 days	18 days	30 days	31 days	32 days	33 days	34 days	Principal
13.562	16.274	27.123	28.027	28.932	29.836	30.740	5500
13.808	16.570	27.616	28.537	29.458	30.378	31.299	5600
14.055	16.866	28.110	29.047	29.984	30.921	31.858	5700
14.301	17.162	28.603	29.556	30.510	31.463	32.416	5800
14.548	17.458	29.096	30.066	31.036	32.005	32.975	5900
14.795	17.753	29.589	30.575	31.562	32.548	33.534	6000
15.041	18.049	30.082	31.085	32.088	33.090	34.093	6100
15.288	18.345	30.575	31.595	32.614	33.633	34.652	6200
15.534	18.641	31.068	32.104	33.140	34.175	35.211	6300
15.781	18.937	31.562	32.614	33.666	34.718	35.770	6400
16.027	19.233	32.055	33.123	34.192	35.260	36.329	6500
16.274	19.529	32.548	33.633	34.718	35.803	36.888	6600
16.521	19.825	33.041	34.142	35.244	36.345	37.447	6700
16.767	20.121	33.534	34.652	35.770	36.888	38.005	6800
17.014	20.416	34.027	35.162	36.296	37.430	38.564	6900
17.260	20.712	34.521	35.671	36.822	37.973	39.123	7000
17.507	21.008	35.014	36.181	37.348	38.515	39.682	7100
17.753	21.304	35.507	36.690	37.874	39.058	40.241	7200
18.000	21.600	36.000	37.200	38.400	39.600	40.800	7300
18.247	21.896	36.493	37.710	38.926	40.142	41.359	7400
18.493	22.192	36.986	38.219	39.452	40.685	41.918	7500
18.740	22.488	37.479	38.729	39.978	41.227	42.477	7600
18.986	22.784	37.973	39.238	40.504	41.770	43.036	7700
19.233	23.079	38.466	39.748	41.030	42.312	43.595	7800
19.479	23.375	38.959	40.258	41.556	42.855	44.153	7900
19.726	23.671	39.452	40.767	42.082	43.397	44.712	8000
19.973	23.967	39.945	41.277	42.608	43.940	45.271	8100
20.219	24.263	40.438	41.786	43.134	44.482	45.830	8200
20.466	24.559	40.932	42.296	43.660	45.025	46.389	8300
20.712	24.855	41.425	42.805	44.186	45.567	46.948	8400
20.959	25.151	41.918	43.315	44.712	46.110	47.507	8500
21.205	25.447	42.411	43.825	45.238	46.652	48.066	8600
21.452	25.742	42.904	44.334	45.764	47.195	48.625	8700
21.699	26.038	43.397	44.844	46.290	47.737	49.184	8800
21.945	26.334	43.890	45.353	46.816	48.279	49.742	8900
22.192	26.630	44.384	45.863	47.342	48.822	50.301	9000
22.438	26.926	44.877	46.373	47.868	49.364	50.860	9100
22.685	27.222	45.370	46.882	48.395	49.907	51.419	9200
22.932	27.518	45.863	47.392	48.921	50.449	51.978	9300
23.178	27.814	46.356	47.901	49.447	50.992	52.537	9400
23.425	28.110	46.849	48.411	49.973	51.534	53.096	9500
23.671	28.405	47.342	48.921	50.499	52.077	53.655	9600
23.918	28.701	47.836	49.430	51.025	52.619	54.214	9700
24.164	28.997	48.329	49.940	51.551	53.162	54.773	9800
24.411	29.293	48.822	50.449	52.077	53.704	55.332	9900

SIMPLE INTEREST TABLES—6%

Principal	60 days	61 days	62 days	63 days	64 days	90 days	93 days
1000	9.863	10.027	10.192	10.356	10.521	14.795	15.288
1100	10.849	11.030	11.211	11.392	11.573	16.274	16.816
1200	11.836	12.033	12.230	12.427	12.625	17.753	18.345
1300	12.822	13.036	13.249	13.463	13.677	19.233	19.874
1400	13.808	14.038	14.268	14.499	14.729	20.712	21.403
1500	14.795	15.041	15.288	15.534	15.781	22.192	22.932
1600	15.781	16.044	16.307	16.570	16.833	23.671	24.460
1700	16.767	17.047	17.326	17.605	17.885	25.151	25.989
1800	17.753	18.049	18.345	18.641	18.937	26.630	27.518
1900	18.740	19.052	19.364	19.677	19.989	28.110	29.047
2000	19.726	20.055	20.384	20.712	21.041	29.589	30.575
2100	20.712	21.058	21.403	21.748	22.093	31.068	32.104
2200	21.699	22.060	22.422	22.784	23.145	32.548	33.633
2300	22.685	23.063	23.441	23.819	24.197	34.027	35.162
2400	23.671	24.066	24.460	24.855	25.249	35.507	36.690
2500	24.658	25.068	25.479	25.890	26.301	36.986	38.219
2600	25.644	26.071	26.499	26.926	27.353	38.466	39.748
2700	26.630	27.074	27.518	27.962	28.405	39.945	41.277
2800	27.616	28.077	28.537	28.997	29.458	41.425	42.805
2900	28.603	29.079	29.556	30.033	30.510	42.904	44.334
3000	29.589	30.082	30.575	31.068	31.562	44.384	45.863
3100	30.575	31.085	31.595	32.104	32.614	45.863	47.392
3200	31.562	32.088	32.614	33.140	33.666	47.342	48.921
3300	32.548	33.090	33.633	34.175	34.718	48.822	50.449
3400	33.534	34.093	34.652	35.211	35.770	50.301	51.978
3500	34.521	35.096	35.671	36.247	36.822	51.781	53.507
3600	35.507	36.099	36.690	37.282	37.874	53.260	55.036
3700	36.493	37.101	37.710	38.318	38.926	54.740	56.564
3800	37.479	38.104	38.729	39.353	39.978	56.219	58.093
3900	38.466	39.107	39.748	40.389	41.030	57.699	59.622
4000	39.452	40.110	40.767	41.425	42.082	59.178	61.151
4100	40.438	41.112	41.786	42.460	43.134	60.658	62.679
4200	41.425	42.115	42.805	43.496	44.186	62.137	64.208
4300	42.411	43.118	43.825	44.532	45.238	63.616	65.737
4400	43.397	44.121	44.844	45.567	46.290	65.096	67.266
4500	44.384	45.123	45.863	46.603	47.342	66.575	68.795
4600	45.370	46.126	46.882	47.638	48.395	68.055	70.323
4700	46.356	47.129	47.901	48.674	49.447	69.534	71.852
4800	47.342	48.132	48.921	49.710	50.499	71.014	73.381
4900	48.329	49.134	49.940	50.745	51.551	72.493	74.910
5000	49.315	50.137	50.959	51.781	52.603	73.973	76.438
5100	50.301	51.140	51.978	52.816	53.655	75.452	77.967
5200	51.288	52.142	52.997	53.852	54.707	76.932	79.496
5300	52.274	53.145	54.016	54.888	55.759	78.411	81.025
5400	53.260	54.148	55.036	55.923	56.811	79.890	82.553

SIMPLE INTEREST TABLES—6%

60 days	61 days	62 days	63 days	64 days	90 days	93 days	Principal
54.247	55.151	56.055	56.959	57.863	81.370	84.082	5500
55.233	56.153	57.074	57.995	58.915	82.849	85.611	5600
56.219	57.156	58.093	59.030	59.967	84.329	87.140	5700
57.205	58.159	59.112	60.066	61.019	85.808	88.668	5800
58.192	59.162	60.132	61.101	62.071	87.288	90.197	5900
59.178	60.164	61.151	62.137	63.123	88.767	91.726	6000
60.164	61.167	62.170	63.173	64.175	90.247	93.255	6100
61.151	62.170	63.189	64.208	65.227	91.726	94.784	6200
62.137	63.173	64.208	65.244	66.279	93.205	96.312	6300
63.123	64.175	65.227	66.279	67.332	94.685	97.841	6400
64.110	65.178	66.247	67.315	68.384	96.164	99.370	6500
65.096	66.181	67.266	68.351	69.436	97.644	100.899	6600
66.082	67.184	68.285	69.386	70.488	99.123	102.427	6700
67.068	68.186	69.304	70.422	71.540	100.603	103.956	6800
68.055	69.189	70.323	71.458	72.592	102.082	105.485	6900
69.041	70.192	71.342	72.493	73.644	103.562	107.041	7000
70.027	71.195	72.362	73.529	74.696	105.041	108.542	7100
71.014	72.197	73.381	74.564	75.748	106.521	110.071	7200
72.000	73.200	74.400	75.600	76.800	108.000	111.600	7300
72.986	74.203	75.419	76.636	77.852	109.479	113.129	7400
73.973	75.205	76.438	77.671	78.904	110.959	114.658	7500
74.959	76.208	77.458	78.707	79.956	112.438	116.186	7600
75.945	77.211	78.477	79.742	81.008	113.918	117.715	7700
76.932	78.214	79.496	80.778	82.060	115.397	119.244	7800
77.918	79.216	80.515	81.814	83.112	116.877	120.773	7900
78.904	80.219	81.534	82.849	84.164	118.356	122.301	8000
79.890	81.222	82.553	83.885	85.216	119.836	123.830	8100
80.877	82.225	83.573	84.921	86.268	121.315	125.359	8200
81.863	83.227	84.592	85.956	87.321	122.795	126.888	8300
82.849	84.230	85.611	86.992	88.373	124.274	128.416	8400
83.836	85.233	86.630	88.027	89.425	125.753	129.945	8500
84.822	86.236	87.649	89.063	90.477	127.233	131.474	8600
85.808	87.238	88.668	90.099	91.529	128.712	133.003	8700
86.795	88.241	89.688	91.134	92.581	130.192	134.532	8800
87.781	89.244	90.707	92.170	93.633	131.671	136.060	8900
88.767	90.247	91.726	93.205	94.685	133.151	137.589	9000
89.753	91.249	92.745	94.241	95.737	134.630	139.118	9100
90.740	92.252	93.764	95.277	96.789	136.110	140.647	9200
91.726	93.255	94.784	96.312	97.841	137.589	142.175	9300
92.712	94.258	95.863	97.348	98.893	139.068	143.704	9400
93.699	95.260	96.822	98.384	99.945	140.548	145.233	9500
94.685	96.263	97.841	99.419	100.997	142.027	146.762	9600
95.671	97.266	98.860	100.455	102.049	143.507	148.290	9700
96.658	98.268	99.879	101.490	103.101	144.986	149.819	9800
97.644	99.271	100.899	102.526	104.153	146.466	151.348	9900

SIMPLE INTEREST TABLES—8%

Principal	15 days	18 days	30 days	31 days	32 days	33 days	34 days
1000	3.288	3.945	6.575	6.795	7.014	7.233	7.452
1100	3.616	4.340	7.233	7.474	7.715	7.956	8.197
1200	3.945	4.734	7.890	8.153	8.416	8.679	8.942
1300	4.274	5.129	8.548	8.833	9.118	9.403	9.688
1400	4.603	5.523	9.205	9.512	9.819	10.126	10.433
1500	4.932	5.918	9.863	10.192	10.521	10.849	11.178
1600	5.260	6.312	10.521	10.871	11.222	11.573	11.923
1700	5.589	6.707	11.178	11.551	11.923	12.296	12.668
1800	5.918	7.101	11.836	12.230	12.625	13.019	13.414
1900	6.247	7.496	12.493	12.910	13.326	13.742	14.159
2000	6.575	7.890	13.151	13.589	14.027	13.466	14.904
2100	6.904	8.285	13.808	14.268	14.729	15.189	15.649
2200	7.233	8.679	14.466	14.948	15.430	15.912	16.395
2300	7.562	9.074	15.123	15.627	16.132	16.636	17.140
2400	7.890	9.468	15.781	16.307	16.833	17.359	17.885
2500	8.219	9.863	16.438	16.986	17.534	18.082	18.630
2600	8.548	10.258	17.096	17.666	18.236	18.805	19.375
2700	8.877	10.652	17.753	18.345	18.937	19.529	20.121
2800	9.205	11.047	18.411	19.025	19.638	20.252	20.866
2900	9.534	11.441	19.068	19.704	20.340	20.975	21.611
3000	9.863	11.836	19.726	20.384	21.041	21.699	22.356
3100	10.192	12.230	20.384	21.063	21.742	22.422	23.101
3200	10.521	12.625	21.041	21.742	22.444	23.145	23.847
3300	10.849	13.019	21.699	22.422	23.145	23.868	24.592
3400	11.178	13.414	22.356	23.101	23.847	24.592	25.337
3500	11.507	13.808	23.014	23.781	24.548	25.315	26.082
3600	11.836	14.203	23.671	24.460	25.249	26.038	26.827
3700	12.164	14.597	24.329	25.140	25.951	26.762	27.573
3800	12.493	14.992	24.986	25.819	26.652	27.485	28.318
3900	12.822	15.386	25.644	26.499	27.353	28.208	29.063
4000	13.151	15.781	26.301	27.178	28.055	28.932	29.808
4100	13.479	16.175	26.959	27.858	28.756	29.655	30.553
4200	13.808	16.570	27.616	28.537	29.458	30.378	31.299
4300	14.137	16.964	28.274	29.216	30.159	31.101	32.044
4400	14.466	17.359	28.932	29.896	30.860	31.825	32.789
4500	14.795	17.753	29.589	30.575	31.562	32.548	33.534
4600	15.123	18.148	30.247	31.255	32.263	33.271	34.279
4700	15.452	18.542	30.904	31.934	32.964	33.995	35.025
4800	15.781	18.937	31.562	32.614	33.666	34.718	35.770
4900	16.110	19.332	32.219	33.293	34.367	35.441	36.515
5000	16.438	19.726	32.877	33.973	35.068	36.164	37.260
5100	16.767	20.121	33.534	34.652	35.770	36.888	38.005
5200	17.096	20.515	34.192	35.332	36.471	37.611	38.751
5300	17.425	20.910	34.849	36.011	37.173	38.334	39.496
5400	17.753	21.304	35.507	36.690	37.874	39.058	40.241

SIMPLE INTEREST TABLES—8%

15 days	18 days	30 days	31 days	32 days	33 days	34 days	Principal
18.082	21.699	36.164	37.370	38.575	39.781	40.986	5500
18.411	22.093	36.822	38.049	39.277	40.504	41.732	5600
18.740	22.488	37.479	38.729	39.978	41.227	42.477	5700
19.068	22.882	38.137	39.408	40.679	41.951	43.222	5800
19.397	23.277	38.795	40.088	41.381	42.674	43.967	5900
19.726	23.671	39.452	40.767	42.082	43.397	44.712	6000
20.055	24.066	40.110	41.447	42.784	44.121	45.458	6100
20.384	24.460	40.767	42.126	43.485	44.844	46.203	6200
20.712	24.855	41.425	42.805	44.186	45.567	46.948	6300
21.041	25.249	42.082	43.485	44.888	46.290	47.693	6400
21.370	25.644	42.740	44.164	45.589	47.014	48.438	6500
21.699	26.038	43.397	44.844	46.290	47.737	49.184	6600
22.027	26.433	44.055	45.523	46.992	48.460	49.929	6700
22.356	26.827	44.712	46.203	47.693	49.184	50.674	6800
22.685	27.222	45.370	46.882	48.395	49.907	51.419	6900
23.014	27.616	46.027	47.562	49.096	50.630	52.164	7000
23.342	28.011	46.685	48.241	49.797	51.353	52.910	7100
23.671	28.405	47.342	48.921	50.499	52.077	53.655	7200
24.000	28.800	48.000	49.600	51.200	52.800	54.400	7300
24.329	29.195	48.658	50.279	51.901	53.523	55.145	7400
24.658	29.589	49.315	50.959	52.603	54.247	55.890	7500
24.986	29.984	49.973	51.638	53.304	54.970	56.636	7600
25.315	30.378	50.630	52.318	54.005	55.693	57.381	7700
25.644	30.773	51.288	52.997	54.707	56.416	58.126	7800
25.973	31.167	51.945	53.677	55.408	57.140	58.871	7900
26.301	31.562	52.603	54.356	56.110	57.863	59.616	8000
26.630	31.956	53.260	55.036	56.811	58.586	60.362	8100
26.959	32.351	53.918	55.715	57.512	59.310	61.107	8200
27.288	32.745	54.575	56.395	58.214	60.033	61.852	8300
27.616	33.140	55.233	57.074	58.915	60.756	62.597	8400
27.945	33.534	55.890	57.753	59.616	61.479	63.342	8500
28.274	33.929	56.548	58.433	60.318	62.203	64.088	8600
28.603	34.323	57.205	59.112	61.019	62.296	64.833	8700
28.932	34.718	57.863	59.792	61.721	63.649	65.578	8800
29.260	35.112	58.521	60.471	62.422	64.373	66.323	8900
29.589	35.507	59.178	61.151	63.123	65.096	67.068	9000
29.918	35.901	59.836	61.830	63.825	65.819	67.814	9100
30.247	36.296	60.493	62.510	64.526	66.542	68.559	9200
30.575	36.690	61.151	63.189	65.227	67.266	69.304	9300
30.904	37.085	61.808	63.868	65.929	67.989	70.049	9400
31.233	37.479	62.466	64.548	66.630	68.712	70.795	9500
31.562	37.874	63.123	65.227	67.332	69.436	71.540	9600
31.890	38.268	63.781	65.907	68.033	70.159	72.285	9700
32.219	38.663	64.438	66.586	68.734	70.882	73.030	9800
32.548	39.058	65.096	67.266	69.436	71.605	73.775	9900

SIMPLE INTEREST TABLES—8%

Principal	60 days	61 days	62 days	63 days	64 days	90 days	93 days
1000	13.151	13.370	13.589	13.808	14.027	19.726	20.384
1100	14.466	14.707	14.948	15.189	15.430	21.699	22.422
1200	15.781	16.044	16.307	16.570	16.833	23.671	24.460
1300	17.096	17.381	17.666	17.951	18.236	25.644	26.499
1400	18.411	18.718	19.025	19.332	19.638	27.616	28.537
1500	19.726	20.055	20.384	20.712	21.041	29.589	30.575
1600	21.041	21.392	21.742	22.093	22.444	31.562	32.614
1700	22.356	22.729	23.101	23.474	23.847	33.534	34.652
1800	23.671	24.066	24.460	24.855	25.249	35.507	36.690
1900	24.986	25.403	25.819	26.236	26.652	37.479	38.729
2000	26.301	26.740	27.178	27.616	28.055	39.452	40.767
2100	27.616	28.077	28.537	28.997	29.458	41.425	42.805
2200	28.932	29.414	29.896	30.378	30.860	43.397	44.844
2300	30.247	30.751	31.255	31.759	32.263	45.370	46.882
2400	31.562	32.088	32.614	33.140	33.666	47.342	48.921
2500	32.877	33.425	33.973	34.521	35.068	49.315	50.959
2600	34.192	34.762	35.332	35.901	36.471	51.288	52.997
2700	35.507	36.099	36.690	37.282	37.874	53.260	55.036
2800	36.822	37.436	38.049	38.663	39.277	55.233	57.074
2900	38.137	38.773	39.408	40.044	40.679	57.205	59.112
3000	39.452	40.110	40.767	41.425	42.082	59.178	61.151
3100	40.767	41.447	42.126	42.805	43.485	61.151	63.189
3200	42.082	42.784	43.485	44.186	44.888	63.123	65.227
3300	43.397	44.121	44.844	45.567	46.290	65.096	67.266
3400	44.712	45.458	46.203	46.948	47.693	67.068	69.304
3500	46.027	46.795	47.562	48.329	49.096	69.041	71.342
3600	47.342	48.132	48.921	49.710	50.499	71.014	73.381
3700	48.658	49.468	50.279	51.090	51.901	72.986	75.419
3800	49.973	50.805	51.638	52.471	53.304	74.959	77.458
3900	51.288	52.142	52.997	53.852	54.707	76.932	79.496
4000	52.603	53.479	54.356	55.233	56.110	78.904	81.534
4100	53.918	54.816	55.715	56.614	57.512	80.877	83.573
4200	55.233	56.153	57.074	57.995	58.915	82.849	85.611
4300	56.548	57.490	58.433	59.375	60.318	84.822	87.649
4400	57.863	58.827	59.792	60.756	61.721	86.795	90.652
4500	59.178	60.164	61.151	62.137	63.123	88.767	92.712
4600	60.493	61.501	62.510	63.518	64.526	90.740	94.773
4700	61.808	62.838	63.868	64.899	65.929	92.712	96.833
4800	63.123	64.175	65.227	66.279	67.332	94.685	98.893
4900	64.438	65.512	66.586	67.660	68.734	96.658	100.953
5000	65.753	66.849	67.945	69.041	70.137	98.630	103.014
5100	67.068	68.186	69.304	70.422	71.540	100.603	105.074
5200	68.384	69.523	70.663	71.803	72.942	102.575	107.134
5300	69.699	70.860	72.022	73.184	74.345	104.548	109.195
5400	71.014	72.197	73.381	74.564	75.748	106.521	111.255

SIMPLE INTEREST TABLES—8%

60 days	61 days	62 days	63 days	64 days	90 days	93 days	Principal
72.329	73.534	74.740	75.945	77.151	108.493	112.110	5500
73.644	74.871	76.099	77.326	78.553	110.466	114.148	5600
74.959	76.208	77.458	78.707	79.956	112.438	116.186	5700
76.274	77.545	78.816	80.088	81.359	114.411	118.225	5800
77.589	78.882	80.175	81.468	82.762	116.384	120.263	5900
78.904	80.219	81.534	82.849	84.164	118.356	122.301	6000
80.219	81.556	82.893	84.230	85.567	120.329	124.340	6100
81.534	82.893	84.252	85.611	86.970	122.301	126.378	6200
82.849	84.230	85.611	86.992	88.373	124.274	128.416	6300
84.164	85.567	86.970	88.373	89.775	126.247	130.455	6400
85.479	86.904	88.329	89.753	91.178	128.219	132.493	6500
86.795	88.241	89.688	91.134	92.581	130.192	134.532	6600
88.110	89.578	91.047	92.515	93.984	132.164	136.570	6700
89.425	90.915	92.405	93.896	95.386	134.137	138.608	6800
90.740	92.252	93.764	95.277	96.789	136.110	140.647	6900
92.055	93.589	95.123	96.658	98.192	138.082	142.685	7000
93.370	94.926	96.482	98.038	99.595	140.055	144.723	7100
94.685	96.263	97.841	99.419	100.997	142.027	146.762	7200
96.000	97.600	99.200	100.800	102.400	144.000	148.800	7300
97.315	98.937	100.559	102.181	103.803	145.973	150.838	7400
98.630	100.274	101.918	103.562	105.205	147.945	152.877	7500
99.945	101.611	103.277	104.942	106.608	149.918	154.915	7600
101.260	102.948	104.636	106.323	108.011	151.890	156.953	7700
102.575	104.285	105.995	107.704	109.414	153.863	158.992	7800
103.890	105.622	107.353	109.085	110.816	155.836	161.030	7900
104.205	106.959	108.712	110.466	112.219	157.808	163.068	8000
106.521	108.296	110.071	111.847	113.622	159.781	165.107	8100
107.836	109.633	111.430	113.227	115.025	161.753	167.145	8200
109.151	110.970	112.789	114.608	116.427	163.726	169.184	8300
110.466	112.307	114.148	115.989	117.830	165.699	171.222	8400
111.781	113.644	115.507	117.370	119.233	167.671	173.260	8500
113.096	114.981	116.866	118.751	120.636	169.644	175.299	8600
114.411	116.318	118.225	120.132	122.038	171.616	177.337	8700
115.726	117.655	119.584	121.512	123.441	173.589	179.375	8800
117.041	118.992	120.942	122.893	124.844	175.562	183.364	8900
118.356	120.329	122.301	124.274	126.247	177.534	185.425	9000
119.671	121.666	123.660	125.655	127.649	179.507	187.485	9100
120.986	123.003	125.019	127.036	129.052	181.479	189.545	9200
122.301	124.340	126.378	128.416	130.455	183.452	191.605	9300
123.616	125.677	127.737	129.797	131.858	185.425	193.666	9400
124.932	127.014	129.096	131.178	133.260	187.397	195.726	9500
126.247	128.351	130.455	132.559	134.663	189.370	197.786	9600
127.562	129.688	131.814	133.940	136.066	191.342	199.847	9700
128.877	131.025	133.173	135.321	137.468	193.315	201.907	9800
130.192	132.362	134.532	136.701	138.871	195.288	203.967	9900

COMPOUND INTEREST TABLES

Periods	1%	1½%	2%	2½%	3%	3½%
1	1.01000	1.01500	1.02000	1.02500	1.03000	1.03500
2	1.02010	1.03023	1.04040	1.05063	1.06090	1.07123
3	1.03030	1.04568	1.06121	1.07689	1.09273	1.10872
4	1.04060	1.06136	1.08243	1.10381	1.12551	1.14752
5	1.05101	1.07728	1.10408	1.13141	1.15927	1.18769
6	1.06152	1.09344	1.12616	1.15969	1.19405	1.22926
7	1.07214	1.10985	1.14869	1.18869	1.22987	1.27228
8	1.08286	1.12649	1.17166	1.21840	1.26677	1.31681
9	1.09369	1.14339	1.19509	1.24886	1.30477	1.36290
10	1.10462	1.16054	1.21899	1.28008	1.34392	1.41060
11	1.11567	1.17795	1.24337	1.31209	1.38423	1.45997
12	1.12683	1.19562	1.26824	1.34489	1.42576	1.51107
13	1.13809	1.21355	1.29361	1.37851	1.46853	1.56396
14	1.14947	1.23176	1.31948	1.41297	1.51259	1.61869
15	1.16097	1.25023	1.34587	1.44830	1.55797	1.67535
16	1.17258	1.26899	1.37279	1.48451	1.60471	1.73399
17	1.18430	1.28802	1.40024	1.52162	1.65285	1.79468
18	1.19615	1.30734	1.42825	1.55966	1.70243	1.85749
19	1.20811	1.32695	1.45681	1.59865	1.75351	1.92250
20	1.22019	1.34686	1.48595	1.63862	1.80611	1.98979
21	1.23239	1.36706	1.51567	1.67958	1.86029	2.05943
22	1.24472	1.38756	1.54598	1.72157	1.91610	2.13151
23	1.25716	1.40838	1.57690	1.76461	1.97359	2.20611
24	1.26973	1.42950	1.60844	1.80873	2.03279	2.28333
25	1.28243	1.45095	1.64061	1.85394	2.09378	2.36324
26	1.29526	1.47271	1.67342	1.90029	2.15659	2.44596
27	1.30821	1.49480	1.70689	1.94780	2.22129	2.53157
28	1.32129	1.51722	1.74102	1.99650	2.28793	2.62017
29	1.33450	1.53998	1.77584	2.04641	2.35657	2.71188
30	1.34785	1.56308	1.81136	2.09757	2.42726	2.80679
31	1.36133	1.58653	1.84759	2.15001	2.50008	2.90503
32	1.37494	1.61032	1.88454	2.20376	2.57508	3.00671
33	1.38869	1.63448	1.92223	2.25885	2.65234	3.11194
34	1.40258	1.65900	1.96068	2.31532	2.73191	3.22086
35	1.41660	1.68388	1.99989	2.37320	2.81386	3.33359
36	1.43077	1.70914	2.03989	2.43253	2.89828	3.45027
37	1.44508	1.73478	2.08068	2.49335	2.98523	3.57102
38	1.45953	1.76080	2.12230	2.55568	3.07478	3.69601
39	1.47412	1.78721	2.16474	2.61957	3.16702	3.82537
40	1.48886	1.81402	2.20804	2.68506	3.26204	3.95926
41	1.50375	1.84123	2.25220	2.75219	3.35990	4.09783
42	1.51879	1.86885	2.29724	2.82100	3.46069	4.24126
43	1.53398	1.89688	2.34319	2.89152	3.56451	4.38970
44	1.54932	1.92533	2.39005	2.96381	3.67145	4.54334
45	1.56481	1.95421	2.43785	3.03790	3.78159	4.70236

COMPOUND INTEREST TABLES

Periods	4%	4½%	5%	6%	7%	8%
1	1.04000	1.04500	1.05000	1.06000	1.07000	1.08000
2	1.08160	1.09203	1.10250	1.12360	1.14490	1.16640
3	1.12486	1.14117	1.15763	1.19102	1.22504	1.25971
4	1.16986	1.19252	1.21551	1.26248	1.31080	1.36049
5	1.21665	1.24618	1.27628	1.33823	1.40255	1.46933
6	1.26532	1.30226	1.34010	1.41852	1.50073	1.58687
7	1.31593	1.36086	1.40710	1.50363	1.60578	1.71382
8	1.36857	1.42210	1.47746	1.59385	1.71819	1.85093
9	1.42331	1.48610	1.55133	1.68948	1.83846	1.99900
10	1.48024	1.55297	1.62889	1.79085	1.96715	2.15893
11	1.53945	1.62285	1.71034	1.89830	2.10485	2.33164
12	1.60103	1.69588	1.79586	2.01220	2.25219	2.51817
13	1.66507	1.77220	1.88565	2.13293	2.40985	2.71962
14	1.73168	1.85194	1.97993	2.26090	2.57853	2.93719
15	1.80094	1.93528	2.07893	2.39656	2.75903	3.17217
16	1.87298	2.02237	2.18287	2.54035	2.95216	3.42594
17	1.94790	2.11338	2.29202	2.69277	3.15882	3.70002
18	2.02582	2.20848	2.40662	2.85434	3.37993	3.99602
19	2.10685	2.30786	2.52695	3.02560	3.61653	4.31570
20	2.19112	2.41171	2.65330	3.20714	3.86968	4.66096
21	2.27877	2.52024	2.78596	3.39956	4.14056	5.03383
22	2.36992	2.63365	2.92526	3.60354	4.43040	5.43654
23	2.46472	2.75217	3.07152	3.81975	4.74053	5.87146
24	2.56330	2.87601	3.22510	4.04893	5.07237	6.34118
25	2.66584	3.00543	3.38635	4.29187	5.42743	6.84848
26	2.77247	3.14068	3.55567	4.54938	5.80735	7.39635
27	2.88337	3.28201	3.73346	4.82235	6.21387	7.98806
28	2.99870	3.42970	3.92013	5.11169	6.64884	8.62711
29	3.11865	3.58404	4.11614	5.41839	7.11426	9.31727
30	3.24340	3.74532	4.32194	5.74349	7.61226	10.06266
31	3.37313	3.91386	4.53804	6.08810	8.14511	10.86767
32	3.50806	4.08998	4.76494	6.45339	8.71527	11.73708
33	3.64838	4.27403	5.00319	6.84059	9.32534	12.67605
34	3.79432	4.46636	5.25335	7.25103	9.97811	13.69013
35	3.94609	4.66735	5.51601	7.68609	10.67658	14.78534
36	4.10393	4.87738	5.79182	8.14725	11.42394	15.96817
37	4.26809	5.09686	6.08141	8.63609	12.22362	17.24563
38	4.43881	5.32622	6.38548	9.15425	13.07927	18.62528
39	4.61637	5.56590	6.70475	9.70351	13.99482	20.11530
40	4.80102	5.81637	7.03999	10.28572	14.97446	21.72453
41	4.99306	6.07810	7.39199	10.90286	16.02268	23.46249
42	5.19278	6.35161	7.76159	11.55704	17.14426	25.33950
43	5.40050	6.63744	8.14967	12.25046	18.34436	27.36665
44	5.61652	6.93612	8.55715	12.98548	19.62846	29.55598
45	5.84118	7.24825	8.98501	13.76461	21.00245	31.92046

PRESENT WORTH TABLES

Periods	1%	1½%	2%	2½%	3%	3½%
1	.990099	.985222	.980392	.975610	.970874	.966184
2	.980296	.970662	.961169	.951814	.942596	.933511
3	.970590	.956317	.942322	.928599	.915142	.901943
4	.960980	.942184	.923845	.905951	.888487	.871442
5	.951466	.928260	.905731	.883854	.862609	.841973
6	.942045	.914542	.887971	.862297	.837484	.813501
7	.932718	.901027	.870560	.841265	.813091	.785991
8	.923483	.887711	.853490	.820747	.789409	.759412
9	.914340	.874592	.836755	.800728	.766417	.733731
10	.905287	.861667	.820348	.781198	.744094	.708919
11	.896324	.848933	.804263	.762145	.722421	.684946
12	.887449	.836387	.788493	.743556	.701380	.661783
13	.878663	.824027	.773032	.725420	.680951	.639404
14	.869963	.811849	.757875	.707727	.661118	.617782
15	.861349	.799851	.743015	.690466	.641862	.596891
16	.852821	.788031	.728446	.673625	.623167	.576706
17	.844377	.776385	.714163	.657195	.605016	.557204
18	.836017	.764912	.700159	.641166	.587395	.538361
19	.827740	.753607	.686431	.625528	.570286	.520156
20	.819544	.742470	.672971	.610271	.553676	.502566
21	.811430	.731498	.659776	.595386	.537549	.485571
22	.803396	.720688	.646839	.580865	.521892	.469151
23	.795442	.710037	.634156	.566697	.506692	.453286
24	.787566	.699544	.621721	.552875	.491934	.437957
25	.779768	.689206	.609531	.539391	.477606	.423147
26	.772048	.679020	.597579	.526235	.463695	.408838
27	.764404	.668986	.585862	.513400	.450189	.395012
28	.756836	.659099	.574375	.500878	.437077	.381654
29	.749342	.649359	.563112	.488661	.424346	.368748
30	.741923	.639762	.552071	.476743	.411987	.356278
31	.734577	.630308	.541246	.465115	.399987	.344230
32	.727304	.620993	.530633	.453771	.388337	.332590
33	.720103	.611816	.520229	.442703	.377026	.321343
34	.712973	.602774	.510028	.431905	.366045	.310476
35	.705914	.593866	.500028	.421371	.355383	.299977
36	.698925	.585090	.490223	.411094	.345032	.289833
37	.692005	.576443	.480611	.401067	.334983	.280032
38	.685153	.567924	.471187	.391285	.325226	.270562
39	.678370	.559531	.461948	.381741	.315753	.261412
40	.671653	.551262	.452890	.372430	.306557	.252572
41	.665003	.543116	.444010	.363347	.297628	.244031
42	.658419	.535089	.435304	.354485	.288959	.235779
43	.651900	.527182	.426769	.345839	.280543	.227806
44	.645446	.519391	.418401	.337404	.272372	.220102
45	.639055	.511715	.410197	.329174	.264439	.212659

PRESENT WORTH TABLES

Periods	4%	4½%	5%	6%	7%	8%
1	.961538	.956938	.952381	.943396	.934579	.925926
2	.924556	.915730	.907029	.889996	.873439	.857339
3	.888996	.876297	.863838	.839619	.816298	.793832
4	.854804	.838561	.822702	.792094	.762895	.735030
5	.821927	.802451	.783526	.747258	.712986	.680583
6	.790314	.767896	.746215	.704960	.666342	.630170
7	.759918	.734828	.710681	.665057	.622750	.583490
8	.730690	.703185	.676839	.627412	.582009	.540269
9	.702587	.672904	.644609	.591898	.543934	.500249
10	.675564	.643928	.613913	.558395	.508349	.463193
11	.649581	.616199	.584679	.526787	.475093	.428883
12	.624597	.589664	.556837	.496969	.444012	.397114
13	.600574	.564272	.530321	.468839	.414964	.367698
14	.577475	.539973	.505068	.442301	.387817	.340461
15	.555264	.516720	.481017	.417265	.362446	.315242
16	.533908	.494469	.458111	.393646	.338735	.291890
17	.513373	.473176	.436297	.371364	.316574	.270269
18	.493628	.452800	.415521	.350344	.295864	.250249
19	.474642	.433302	.395734	.330513	.276508	.231712
20	.456387	.414643	.376889	.311805	.258419	.214548
21	.438834	.396787	.358942	.294155	.241513	.198656
22	.421955	.379701	.341849	.277505	.225713	.183940
23	.405726	.363350	.325571	.261797	.210947	.170315
24	.390121	.347703	.310068	.246978	.197147	.157699
25	.375117	.332731	.295303	.232999	.184249	.146018
26	.360689	.318402	.281241	.219810	.172195	.135202
27	.346817	.304691	.267848	.207368	.160930	.125187
28	.333477	.291571	.255094	.195630	.150402	.115914
29	.320651	.279015	.242946	.184557	.140563	.107327
30	.308319	.267000	.231377	.174110	.131367	.099377
31	.296460	.255502	.220359	.164255	.122773	.092016
32	.285058	.244500	.209866	.154957	.114741	.085200
33	.274094	.233971	.199872	.146186	.107235	.078889
34	.263552	.223896	.190355	.137911	.100219	.073045
35	.253415	.214254	.181290	.130105	.093663	.067634
36	.243669	.205028	.172657	.122741	.087535	.062625
37	.234297	.196199	.164436	.115793	.081809	.057986
38	.225285	.187750	.156605	.109239	.076457	.053690
39	.216621	.179665	.149148	.103055	.071455	.049713
40	.208289	.171929	.142046	.097222	.066780	.046031
41	.200278	.164525	.135282	.091709	.062412	.042621
42	.192575	.157440	.128840	.086527	.058329	.039464
43	.185168	.150661	.122704	.081630	.054513	.036541
44	.178046	.144173	.116861	.077009	.050946	.033834
45	.171198	.137964	.111296	.072650	.047613	.031328

No.	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	10 346
127	10 380	10 415	10 449	10 483	10 517	10 551	10 585	10 619	10 653	10 687
128	10 721	10 755	10 789	10 823	10 857	10 890	10 924	10 958	10 992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	11 361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	11 727	11 760	11 793	11 826	11 860	11 893	11 926	11 959	11 992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	12 352
133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
134	12 710	12 743	12 775	12 808	12 840	12 872	12 905	12 937	12 969	13 001
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136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
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142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
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145	16 137	16 167	16 197	16 277	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
149	17 319	17 348	17 377	17 406	17 435	17 464	17 493	17 522	17 551	17 580
150	17 609	17 638	17 667	17 696	17 725	71 754	17 782	17 811	17 840	17 869

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152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
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156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
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159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
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164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
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171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
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179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
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183	26 245	26 269	26 293	26 316	26 340	26 364	26 387	26 411	26 435	26 458
184	26 482	26 505	26 529	26 553	26 576	26 600	26 623	26 647	26 670	26 694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
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187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
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189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852
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191	28 103	28 126	28 149	28 171	28 194	28 217	28 240	28 262	28 285	28 307
192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
194	28 780	28 803	28 825	28 847	28 870	28 892	28 914	28 937	28 959	28 981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
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205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
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210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
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214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
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226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
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229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
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243	38 561	38 578	38 596	38 614	38 632	38 650	38 668	38 686	38 703	38 721
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246	39 094	39 111	39 129	39 146	39 164	39 182	39 199	39 217	39 235	39 252
247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950

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253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
255	40 654	40 671	40 688	40 705	40 722	40 739	40 756	40 773	40 790	40 807
256	40 824	40 841	40 858	40 875	40 892	40 909	40 926	40 943	40 960	40 976
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258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
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264	42 160	42 177	42 193	42 210	42 226	42 243	42 259	42 275	42 292	42 308
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266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
267	42 651	42 667	42 684	42 700	42 716	42 732	42 749	42 765	42 781	42 797
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269	42 975	42 991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
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271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
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274	43 775	43 791	43 807	43 823	43 838	43 854	43 870	43 886	43 902	43 917
275	43 933	43 949	43 965	43 981	43 996	44 012	44 028	44 044	44 059	44 075
276	44 091	44 107	44 122	44 138	44 154	44 170	44 185	44 201	44 217	44 232
277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
279	44 560	44 576	44 592	44 607	44 623	44 638	44 654	44 669	44 685	44 700
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281	44 871	44 886	44 902	44 917	44 932	44 948	44 963	44 979	44 994	45 010
282	45 025	45 040	45 056	45 071	45 086	45 102	45 117	45 133	45 148	45 163
283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
284	45 332	45 347	45 362	45 378	45 393	45 408	45 423	45 439	45 454	45 469
285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
286	45 637	45 652	45 667	45 682	45 697	45 712	45 728	45 743	45 758	45 773
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288	45 939	45 954	45 969	45 984	46 000	46 015	46 030	46 045	46 060	46 075
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292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 672
293	46 687	46 702	46 716	46 731	46 746	46 761	46 776	46 790	46 805	46 820
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295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
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298	47 422	47 436	47 451	47 465	47 480	47 494	47 509	47 524	47 538	47 553
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300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842

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311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
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317	50 106	50 120	50 133	50 147	50 161	50 174	50 188	50 202	50 215	50 229
318	50 243	50 256	50 270	50 284	50 297	50 311	50 325	50 338	50 352	50 365
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320	50 515	50 529	50 542	50 556	50 569	50 583	50 596	50 610	50 623	50 637
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334	52 375	52 388	52 401	52 414	52 427	52 440	52 453	52 466	52 479	52 492
335	52 504	52 517	52 530	52 543	52 556	52 569	52 582	52 595	52 608	52 621
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341	53 275	53 288	53 301	53 314	53 326	53 339	53 352	53 364	53 377	53 390
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350	54 407	54 419	54 432	54 444	54 456	54 469	54 481	54 494	54 506	54 518

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527	72 181	72 189	72 198	72 206	72 214	72 222	72 230	72 239	72 247	72 255
528	72 263	72 272	72 280	72 288	72 296	72 304	72 313	72 321	72 329	72 337
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546	73 719	73 727	73 735	73 743	73 751	73 759	73 767	73 775	73 783	73 791
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548	73 878	73 886	73 894	73 902	73 910	73 918	73 926	73 933	73 941	73 949
549	73 957	73 965	73 973	73 981	73 989	73 997	74 005	74 013	74 020	74 028
550	74 036	74 044	74 052	74 060	74 068	74 076	74 084	74 092	74 099	74 107

No.	0	1	2	3	4	5	6	7	8	9
550	74 036	74 044	74 052	74 060	74 068	74 076	74 084	74 092	74 099	74 107
551	74 115	74 123	74 131	74 139	74 147	74 155	74 162	74 170	74 178	74 186
552	74 194	74 202	74 210	74 218	74 225	74 233	74 241	74 249	74 257	74 265
553	74 273	74 280	74 288	74 296	74 304	74 312	74 320	74 327	74 335	74 343
554	74 351	74 359	74 367	74 374	74 382	74 390	74 398	74 406	74 414	74 421
555	74 429	74 437	74 445	74 453	74 461	74 468	74 476	74 484	74 492	74 500
556	74 507	74 515	74 523	74 531	74 539	74 547	74 554	74 562	74 570	74 578
557	74 586	74 593	74 601	74 609	74 617	74 624	74 632	74 640	74 648	74 656
558	74 663	74 671	74 679	74 687	74 695	74 702	74 710	74 718	74 726	74 733
559	74 741	74 749	74 757	74 764	74 772	74 780	74 788	74 796	74 803	74 811
560	74 819	74 827	74 834	74 842	74 850	74 858	74 865	74 873	74 881	74 889
561	74 896	74 904	74 912	74 920	74 927	74 935	74 943	74 950	74 958	74 966
562	74 974	74 981	74 989	74 997	75 005	75 012	75 020	75 028	75 035	75 043
563	75 051	75 059	75 066	75 074	75 082	75 089	75 097	75 105	75 113	75 120
564	75 128	75 136	75 143	75 151	75 159	75 166	75 174	75 182	75 189	75 197
565	75 205	75 213	75 220	75 228	75 236	75 243	75 251	75 259	75 266	75 274
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567	75 358	75 366	75 374	75 381	75 389	75 397	75 404	75 412	75 420	75 427
568	75 435	75 442	75 450	75 458	75 465	75 473	75 481	75 488	75 496	75 504
569	75 511	75 519	75 526	75 534	75 542	75 549	75 557	75 565	75 572	75 580
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571	75 664	75 671	75 679	75 686	75 694	75 702	75 709	75 717	75 724	75 732
572	75 740	75 747	75 755	75 762	75 770	75 778	75 785	75 793	75 800	75 808
573	75 815	75 823	75 831	75 838	75 846	75 853	75 861	75 868	75 876	75 884
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577	76 118	76 125	76 133	76 140	76 148	76 155	76 163	76 170	76 178	76 185
578	76 193	76 200	76 208	76 215	76 223	76 230	76 238	76 245	76 253	76 260
579	76 268	76 275	76 283	76 290	76 298	76 305	76 313	76 320	76 328	76 335
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581	76 418	76 425	76 433	76 440	76 448	76 455	76 462	76 470	76 477	76 485
582	76 492	76 500	76 507	76 515	76 522	76 530	76 537	76 545	76 552	76 559
583	76 567	76 574	76 582	76 589	76 597	76 604	76 612	76 619	76 626	76 634
584	76 641	76 649	76 656	76 664	76 671	76 678	76 686	76 693	76 701	76 708
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587	76 864	76 871	76 879	76 886	76 893	76 901	76 908	76 916	76 923	76 930
588	76 938	76 945	76 953	76 960	76 967	76 975	76 982	76 989	76 997	77 004
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591	77 159	77 166	77 173	77 181	77 188	77 195	77 203	77 210	77 217	77 225
592	77 232	77 240	77 247	77 254	77 262	77 269	77 276	77 283	77 291	77 298
593	77 305	77 313	77 320	77 327	77 335	77 342	77 349	77 357	77 364	77 371
594	77 379	77 386	77 393	77 401	77 408	77 415	77 422	77 430	77 437	77 444
595	77 452	77 459	77 466	77 474	77 481	77 488	77 495	77 503	77 510	77 517
596	77 525	77 532	77 539	77 546	77 554	77 561	77 568	77 576	77 583	77 590
597	77 597	77 605	77 612	77 619	77 627	77 634	77 641	77 648	77 656	77 663
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599	77 743	77 750	77 757	77 764	77 772	77 779	77 786	77 793	77 801	77 808
600	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880

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600	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880
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602	77 960	77 967	77 974	77 981	77 988	77 996	78 003	78 010	78 017	78 025
603	78 032	78 039	78 046	78 053	78 061	78 068	78 075	78 082	78 089	78 097
604	78 104	78 111	78 118	78 125	78 132	78 140	78 147	78 154	78 161	78 168
605	78 176	78 183	78 190	78 197	78 204	78 211	78 219	78 226	78 233	78 240
606	78 247	78 254	78 262	78 269	78 276	78 283	78 290	78 297	78 305	78 312
607	78 319	78 326	78 333	78 340	78 347	78 355	78 362	78 369	78 376	78 383
608	78 390	78 398	78 405	78 412	78 419	78 426	78 433	78 440	78 447	78 455
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610	78 533	78 540	78 547	78 554	78 561	78 569	78 576	78 583	78 590	78 597
611	78 604	78 611	78 618	78 625	78 633	78 640	78 647	78 654	78 661	78 668
612	78 675	78 682	78 689	78 696	78 704	78 711	78 718	78 725	78 732	78 739
613	78 746	78 753	78 760	78 767	78 774	78 781	78 789	78 796	78 803	78 810
614	78 817	78 824	78 831	78 838	78 845	78 852	78 859	78 866	78 873	78 880
615	78 888	78 895	78 902	78 909	78 916	78 923	78 930	78 937	78 944	78 951
616	78 958	78 965	78 972	78 979	78 986	78 993	79 000	79 007	79 014	79 021
617	79 029	79 036	79 043	79 050	79 057	79 064	79 071	79 078	79 085	79 092
618	79 099	79 106	79 113	79 120	79 127	79 134	79 141	79 148	79 155	79 162
619	79 169	79 176	79 183	79 190	79 197	79 204	79 211	79 218	79 225	79 232
620	79 239	79 246	79 253	79 260	79 267	79 274	79 281	79 288	79 295	79 302
621	79 309	79 316	79 323	79 330	79 337	79 344	79 351	79 358	79 365	79 372
622	79 379	79 386	79 393	79 400	79 407	79 414	79 421	79 428	79 435	79 442
623	79 449	79 456	79 463	79 470	79 477	79 484	79 491	79 498	79 505	79 511
624	79 518	79 525	79 532	79 539	79 546	79 553	79 560	79 567	79 574	79 581
625	79 588	79 595	79 602	79 609	79 616	79 623	79 630	79 637	79 644	79 650
626	79 657	79 664	79 671	79 678	79 685	79 692	79 699	79 706	79 713	79 720
627	79 727	79 734	79 741	79 748	79 754	79 761	79 768	79 775	79 782	79 789
628	79 796	79 803	79 810	79 817	79 824	79 831	79 837	79 844	79 851	79 858
629	79 865	79 872	79 879	79 886	79 893	79 900	79 906	79 913	79 920	79 927
630	79 934	79 941	79 948	79 955	79 962	79 969	79 975	79 982	79 989	79 996
631	80 003	80 010	80 017	80 024	80 030	80 037	80 044	80 051	80 058	80 065
632	80 072	80 079	80 085	80 092	80 099	80 106	80 113	80 120	80 127	80 134
633	80 140	80 147	80 154	80 161	80 168	80 175	80 182	80 188	80 195	80 202
634	80 209	80 216	80 223	80 229	80 236	80 243	80 250	80 257	80 264	80 271
635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	80 346	80 353	80 359	80 366	80 373	80 380	80 387	80 393	80 400	80 407
637	80 414	80 421	80 428	80 434	80 441	80 448	80 455	80 462	80 468	80 475
638	80 482	80 489	80 496	80 502	80 509	80 516	80 523	80 530	80 536	80 543
639	80 550	80 557	80 564	80 570	80 577	80 584	80 591	80 598	80 604	80 611
640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
641	80 686	80 693	80 699	80 706	80 713	80 720	80 726	80 733	80 740	80 747
642	80 754	80 760	80 767	80 774	80 781	80 787	80 794	80 801	80 808	80 814
643	80 821	80 828	80 835	80 841	80 848	80 855	80 862	80 868	80 875	80 882
644	80 889	80 895	80 902	80 909	80 916	80 922	80 929	80 936	80 943	80 949
645	80 956	80 963	80 969	80 976	80 983	80 990	80 996	81 003	81 010	81 017
646	81 023	81 030	81 037	81 043	81 050	81 057	81 064	81 070	81 077	81 084
647	81 090	81 097	81 104	81 111	81 117	81 124	81 131	81 137	81 144	81 151
648	81 158	81 164	81 171	81 178	81 184	81 191	81 198	81 204	81 211	81 218
649	81 224	81 231	81 238	81 245	81 251	81 258	81 265	81 271	81 278	81 285
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351

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650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
651	81 358	81 365	81 371	81 378	81 385	81 391	81 398	81 405	81 411	81 418
652	81 425	81 431	81 438	81 445	81 451	81 458	81 465	81 471	81 478	81 485
653	81 491	81 498	81 505	81 511	81 518	81 525	81 531	81 538	81 544	81 551
654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	81 690	81 697	81 704	81 710	81 717	81 723	81 730	81 737	81 743	81 750
657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
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660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
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665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
668	82 478	82 484	82 491	82 497	82 504	82 510	82 517	82 523	82 530	82 536
669	82 543	82 549	82 556	82 562	82 569	82 575	82 582	82 588	82 595	82 601
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671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	83 065	83 072	83 078	83 085	83 091	83 097	83 104	83 110	83 117
678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	87 499
684	83 506	83 512	83 518	83 525	83 531	83 537	83 544	83 550	83 556	83 563
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	83 632	83 639	83 645	83 651	83 658	83 664	83 670	83 677	83 683	83 689
687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
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699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	84 491	84 497	84 504
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566

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700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
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707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
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712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
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734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
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741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558

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750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076
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761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
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764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
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767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
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769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
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773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	89 193	89 198	89 204
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782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
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784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
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786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
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793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
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799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 304
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802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
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804	90 526	90 531	90 536	90 542	90 547	90 553	90 558	90 563	90 569	90 574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 084	91 089	91 094	91 100	91 105	91 110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
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818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376
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821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 603	91 609	91 614	91 619	91 624	91 630	91 635	91 640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
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827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
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831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
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834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
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839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
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848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988

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851	92 993	92 998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	93 054	93 059	93 064	93 069	93 075	93 080	93 085	93 090
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854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
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860	93 450	93 455	93 460	93 465	93 470	93 475	93 480	93 485	93 490	93 495
861	93 500	93 505	93 510	93 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
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874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
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882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
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885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
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897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
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899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468

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903	95 569	95 574	95 578	95 583	95 588	95 593	94 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
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915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
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917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
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921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
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925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
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932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
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940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
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945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
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949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813

No.	0	1	2	3	4	5	6	7	8	9
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951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
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953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
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959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039

ANSWERS

Exercise Ia. Page 11. **2.** 28450000. **3.** 1935; 5142.6; 31410; 1782; 2961; 1675. **4.** 2158100; 92428125; 245112500; $3201666\frac{2}{3}$. **5.** Quotient 249, Remainder 57. **6.** (a) 48498, 31591, 49372, 383295; (b) 3817350, 686070, 2458573, 40634. **7.** 80° 27′.

Exercise Ib. Page 12. **1.** (a) 12; (b) 14; (c) 25. **2.** 127; 239; 1; 113. **3.** (a) $2^2.3^3.7$; (b) $2^3.3^3.7.17$; (c) $2^9.3^2$. **4.** 1 ft. 6 in.; 6 in.; 1 gal. **5.** (a) 14256; (b) 4256532; (c) 12600. **6.** 121. **7.** 5 **8.** 10; 30; 24; 20; 15. **9.** 34. **10.** 66 ft.

Exercise IIa. Page 19. **1.** $\frac{8}{9}$. **2.** $\frac{29}{120}$. **3.** $\frac{3}{4}$. **4.** (a) 2156 in.; (b) 301 pts.; (c) 498462 sec.; (d) 620412 sq. in.; (e) 69196 oz. **5.** (a) 537 bush. 2 qts.; (b) $\frac{7}{16}$; (c) 5 mi. 1200 yds.; (d) 1 wk. 1 dy. 14 hrs. 34 min. 53 sec. **6.** £246 11s. 6.08d. **7.** $8033.04. **8.** £140 9s. 3.78d. **9.** $29\frac{1}{6}$. **10.** $\frac{4357}{10890}$. **11.** 253.18 gr. **12.** 28 ft.

Exercise IIb. Page 20. **1.** 10 yds. 1 ft. 10 in. **2.** 1 yd. 1 ft. 6 in. **3.** £28 0s. 8d. **4.** 78 yds. 4 in. **5.** 2 cwt. 78 lbs. 12 oz. **6.** 3 mi. 1100 yds. 1 ft. **7.** 31. **8.** 100 T. 8 cwt. 16 lbs. **9.** 1864. **10.** 299+. **11.** 104+ gal. **12.** 137896. **13.** $2\frac{3}{5}$ ac. **14.** 44. **15.** 105. **16.** $316\frac{4}{5}$. **17.** 2 T. 4 cwt. **18.** $75.37. **19.** 52800 yds. **20.** 4 hrs. 8 min. $31\frac{1}{5}$ sec.

Exercise IIc. Page 21. **1.** (a) 635.016 m. = 6350.16 dm. = 63501.6 cm. = 635016 mm. = 635016000 um. = 63.5016 Dm. = 6.35016 Hm. = .635016 Km. = .0635016 Mm. = .000635016 Mgm.; (d) 96.45 g. = 964.5 dg. = 9645 cg. = 96450 mg. = 9.645 Dg. = .9645 Hg. = .09645 Kg. = .00009645 T.; (g) 264.591 l. = 2645.91 dl. = 26459.1 cl. = 264591ml. = 26.4591 Dl. = 2.64591 Hl. = .264591 Kl. **2.** (a) 7.87 in., 8.75 yds., 7.6 mi., 61.8 ac., $11\frac{1}{4}$ qts., 22.55 lbs.; (b) 25.64 cm., 15.2 m., 20 Km., 2.02 Ha., $1\frac{2}{3}$ l., 2265 l., 6.4 Kg., 113.4 g. **3.** 60. **4.** 26; 16 mm. **5.** 3.0 in.; 16.5 in. **6.** 778 mm. **7.** 8000. **8.** (a) 4.104 Dm; (b) 6840.5 cm. **9.** 946×10^{10}. **10.** 22343.75. **11.** 360. **12.** 343 m. **13.** 11.43 Hl. **14.** 20. **15.** 909. **16.** 75.022 Km. **17.** (a) 482.5 g.; (b) 14.2008 Kg., (c) 1448.86 Kg. **18.** 28.32 l. **19.** 511 yds. **20.** 16.03.

Exercise IIIa. Page 28. **1.** (a) $21\frac{11}{20}$; (b) $1\frac{4}{9}$; (c) $\frac{223}{480}$; (d) $\frac{173}{575}$; (e) $2\frac{107}{465}$. **2.** $3\frac{3}{10}$. **3.** $2\frac{321}{886}$. **4.** $\frac{2}{7}$. **5.** $\frac{11}{30}$. **6.** 294. **7.** $\frac{61}{144}$. **8.** $5\frac{33}{40}$. **9.** $5\frac{17}{21}$. **10.** 252 sq. in. **11.** (1) $38\frac{4}{5}$ in.; (2) $58\frac{151}{160}$ sq. in.;

$27\frac{3}{4}$ cu. in. **12.** $8\frac{127}{192}$ cu. in. **13.** $14\frac{43}{100}$. **14.** $12\frac{31}{40}$. **15.** $\frac{6}{7}$. **16.** $\frac{3}{5}$ in. **17.** $\frac{4}{5}$ sq. in. **18.** $12\frac{2}{5}$. **19.** $\frac{73}{184}$. **20.** 186 mi. **21.** $\frac{8}{57}$. **22.** $\frac{4}{11}$.

Exercise IIIb. Page 30. **1.** 2. **2.** $\frac{8}{11}$. **3.** (a) $\frac{3}{4}$; (b) $\frac{11}{24}$; (c) $2\frac{2}{3}$; (d) 3; (e) $1\frac{89}{207}$; (f) $\frac{22}{105}$. **4.** \$22.50. **5.** A, $\frac{7}{30}$ mi. **6.** $6\frac{9}{26}$ mi. **7.** 6th. **8.** 20, 1 in. **9.** \$3640. **10.** $\frac{2}{3}$. **11.** \$76.74. **12** $33\frac{3}{22}$ ft. **13.** $\frac{5}{8}$. **14.** \$22.50. **15.** $52\frac{1}{2}$ min. **16.** $3\frac{1}{4}$ mi. **17.** 1 gal. per sec.

Exercise IVa. Page 41. **1.** Three, and three-hundredths; Six-hundredths; Four, and eighty-nine hundredths; Three hundred and forty-seven thousandths; Eight thousandths; Four hundred and thirty-eight, and four hundred and thirty-eight thousandths; Seven ten thousandths Forty-five ten thousandths; Three millionths; Four thousand, and four ten thousandths **2.** .989, .9, .899, .5, 498, .14, .0624, .0598, .0119. **3.** $\frac{3}{50}$, $\frac{13}{20}$, $\frac{1}{200}$, $\frac{3}{800}$, $\frac{15}{16}$, $\frac{3}{25}$, $\frac{1}{4000}$, $\frac{17}{400}$. **4.** .96, .7778, .875, .024, 0156, .0008, 4.9408, 2.1232, .6667, .4545, .4054, .3273, .5926, .3924. **5.** (a) 6635.6668; (b) 3790.96245. **6.** (a) 1.2236, 66.1104, 47.7422, 3.36258; (b) 4.9562, 8.91265, 8.837, .0637. **7.** $\frac{37}{99}$, $\frac{5}{111}$, $\frac{62051}{9990}$, $\frac{421}{990}$, $\frac{28129}{9000}$, $\frac{2713}{19980}$. **8.** (a) 16.9673; (b) 1.6709; (c) 31.6740. **9.** 1.954. **10.** 9 mm.

Exercise IVb. Page 42. **1.** (1) 341.7; (2) 42.63; (3) 65.6; (4) 370.4; (5) 103900. **2.** 6.452, 1697.545. **3.** 1.52098. **4.** 15, 76 cts. **5.** 29.84246. **6.** (1) 4080; (2) .0587; (3) .036497; (4) .09008; (5) 64.5426. **7.** 75.625. **8.** 2.977. **9.** 2.3125. **10.** 2.1144. **11.** 203.898; (1) 2038.98; (2) .203898; (3) 20.3898; (4) 20.3898. **12.** 261.059. **13.** 6.452. **14.** .286. **15.** .31207. **16.** .02916. **17.** 2897, 7 mm. **18.** .0018. **19.** 282.25. **20.** 298.2 cu. in.

Exercise IVc. Page 44. **1.** 37.4. **2.** (a) 193.734; (b) .178; (c) 23; (d) 2.55. **3.** (a) 5.59; (b) 5.746; (c) .695. **4.** (a) 18.20; (b) 20.0; (c) .327; (d) 646; (e) 2345.087; (f) .0755; (g) 1170.3484. **6.** (a) 1.6512; (b) .7548; (c) .62168; (d) .637; (e) 1.74; (f) 1.7045; (g) .07526. **7.** 26.0 f. **8.** \$648.23, \$492.03, \$421.74. **9.** 369 cm. **10.** 31.6 sq. in. **11.** 12. **12.** 13.36. **13.** 28.18 in. **14.** 5558 sq. dm.

Exercise Va. Page 50. **1.** \$3150, \$4200. **2.** $11\frac{1}{4}$ in. **3.** 20 in., 35 in. **4.** 5 : 12. **5.** 6 : 5. **6.** 30, 54. **7.** 4 : 1, 9 : 10, 3 : 8, 9 : 2, 1 : 12, 2 : 3, 13 : 8, 9 : 1. **8.** 140 lbs. **9.** 73 yds. 1 ft. **10.** $183\frac{1}{3}$ ac. **11.** 55 lbs. 9 oz., 6 lbs. 15 oz. **12.** \$120, \$60, \$20. **13.** 20 lbs., 15 lbs., 10 lbs. **14.** 8 : 45, 2 : 9, 3 : 5. **15.** .81, 3.24, 7.29 and 12.96 sq. in.

16. 2.4. **17.** 100 ft. **18.** 50 lbs., 100 lbs. **19.** 6 in. **20.** $21\frac{3}{7}$ lbs., $35\frac{5}{7}$ lbs., $42\frac{6}{7}$ lbs. **21.** 457.78 c.c. **22.** 1700 lbs. **23.** $1\frac{4}{5}$ cu. ft., $2\frac{4}{25}$ cu. ft.

Exercise Vb. Page **53.** **1.** 400. **2.** 9 in. **3.** $\frac{1}{16}$ sq. in. **4.** 12 mi. = 1 in. **5.** 36.864. **6.** 12 mi. = 1 in. **7.** (a) 92093760 mi.; (b) 855.360 mi. **8.** (a) $2\frac{2}{9}$ dys.; (b) 4; (c) $486. **9.** (a) $540; (b) 9 hrs.; (c) 15 dys. **10.** 240 days longer. **11.** 150. **12.** $10400, $7000. **13.** $6\frac{1}{4}$ mi., 77 min. **14.** $3\frac{3}{4}$ hr. 35 mi. per hr. **15.** 180. **16.** 48 da, 30 da. **17.** $4\frac{2}{3}$. **18.** $2\frac{1}{2}$ da. **19.** 16. **20.** 12. **21.** 28. **22.** $350. **23.** 7.

Exercise VI. Page **60.** **1.** 52° 30′. **2.** West, 22° 30′. **3.** 7 min. 20 sec. **4.** 13° 38′ 15″ E. **5.** 1 hr. 54 min. 28 sec. p.m. **6.** East, 58° 25′, 10″, W. **7.** 2 hrs. 50 min. 14.8 sec. p.m. **8.** 8 o'clock. **9.** 11 hrs. 49 min. 30 sec. a.m. **10.** 8 hrs. 47 min. 31 sec. a.m. **11.** 16° 5′ W. **12.** 7 hrs. 59 min. 38.3 sec. a.m. **13.** 9 hrs. 29 min. 52 sec. p.m. **14.** 79° 24′ 18″ W. **15.** On the time meridians, midway between the time meridians, $\frac{1}{2}$ hr., West half, East half. **16.** 89° 12′, 90°, 6 a.m. **17.** 1 hr. 22 min. **18.** 4 hrs. **19.** 18 hrs. **20.** 43° 9′.

Exercise VIIa. Page **68.** **1.** 17, 21, 37, 59, 73, 78, 86, 91, 123, 234, 304, 609, 895, 719, 653. **2.** 42, 33, 63, 78, 225, 84. **3.** $1\frac{1}{4}$, $1\frac{1}{12}$, $1\frac{2}{15}$, $2\frac{1}{6}$, $3\frac{1}{8}$. **5.** 1.41, 2.24, 4.47, 14.14, 1.73, 5.48, 6.93, 3.61, 11.40, 13.49, 20.47, 73.97, 80.49, 89.24, 99.92. **6.** 5.7, 1.19, 3.58, 61.5, 10.01, .65, .037, .131, .0907, .027, 10.17, .773. **7.** .58, 1.12, .61, .77, 2.10, 3.91, 2.81. **8.** .92, .65, .83, 1.83, 2.10, 1.22, 2.58, .67, .84, .73. **9.** 2838.56, 166.687, 527.11, 17.54, 55.476, 1.7320508, 1.41421, 2.236..

Exercise VIIb. Page **70.** **3.** 6.32. **4.** 2.484. **5.** 12, 9, 21. **6.** 1.5. **7.** 98.39, 695.70, 1244.51. **8.** 880. **9.** Side 10.4 in. **10.** 6 ft. × 18 ft. **11.** 240. **12.** 20 in. × 35 in. **13.** 9 ft. × 12 ft. × 15 ft.

Exercise VIIIa. Page **81.** **1.** 6.5 in. **2.** 19.18 mm. **3.** 15 ft. 5 in. **4.** $\frac{a}{2}\sqrt{3}$ in. **5.** 11.31 in. **6.** 70 yds. **7.** 200. **8.** 207.4 ft. **9.** 10 in. **10.** $2\frac{1}{2}$ mi. **11.** 720. **12.** 48 ft. 10 in. **13.** 775.2 yds. **14.** (1) $654\frac{6}{11}$ degrees; (2) $36\frac{4}{11}$. **15.** $9\frac{1}{3}$ in. **16.** 21.6 in. **17.** 12 in. **18.** $5\frac{13}{15}$ in. **19.** 57.7 cm. **20.** 44 in. **21.** 6.72 ft. **22.** 32 in. **23.** 2 ft. 10.6 in. **24.** 89 ft. 5 in. **25.** 52 ft.

Exercise VIIIb. **Page 83.** **1.** \$1800. **2.** 1.205. **3.** 346.5 sq. m., 38.5 sq. in., 13.86 sq. ft. **5.** $2\frac{13}{32}$ sq. ft. **6.** 693 sq. in. **7.** $3\sqrt{3} : 4$. **8.** \$64.26. **9.** 638.82 sq. cm. **10.** 72.42 sq. in. **11.** 52.6 yds. **12.** 88 ft. **13.** 1.62 ac. **14.** .43 sq. m. **15.** 129.22 sq. cm. **17.** 173.2 sq. cm. **18.** 2.43. **19.** \$341.79. **20.** 12.56 sq. m., 19.625 sq. m., 28.26 sq. m. **21.** 2.92 ohms.

Exercise VIIIc. **Page 85.** **1.** $86\frac{2}{5}$ cts. **2.** 6.452 sq. cm. **3.** 60 sq. ft. **4.** 528000. **5.** 330 sq. yds. 1 sq. ft. 36 sq. in. **6.** 15 ft. × 15 ft. **7.** 60 rods. **8.** \$855.04. **9.** \$477.27. **10.** 42 ft. **11.** $19\frac{1}{8}$ ac. **12.** 20 in. **13.** \$770. **14.** .22 Km. **15.** 15 ft., $18\frac{3}{4}$ ft. **16.** 24 rds. × 30 rds. **17.** \$329.85. **18.** 810 ac. **19.** 2088 sq. ft. **20.** 97 sq. ft. **21.** $32\frac{7}{10}$ ac. **22.** \$1383.12. **23.** 36 men to a side. **24.** 12 bush. **25.** \$19.44.

Exercise VIIId. **Page 89.** **1.** $1163\frac{1}{4}$ sq. in. **2.** 7.8 cm., 740.88 sq. cm. **3.** 2112 sq. yds. **4.** $2\frac{3}{4}$ sq. ft. **5.** $2\frac{1}{3}$ sq. ft. **6.** 91 cts. **7.** 4.77 cm. **9.** 1886.5 sq. cm. **10.** $2 : 2 : \sqrt{2}$. **11.** 1698 sq. yds. **12.** \$80. **13.** 22 sq. ft. **14.** $\sqrt{\pi} : \sqrt{6}$. **15.** 188.5 sq. ft. **16.** 146.14 sq. in. **17.** 405.8 sq. in. **18.** 4 : 1. **19.** 1277.1 sq. in. **20.** 560 mi. to 1 inch. **21.** $2\frac{3}{4}$ in.

Exercise IXa. **Page 97.** **1.** 512. **2.** $192\frac{6}{7}$ Kg. **3.** $2\frac{43}{66}$ lbs. **4.** $13612\frac{1}{2}$ tons. **5.** $5\frac{1}{6}$ sq. ft. **6.** 91125 tons. **7.** 6 in., 15 min. **8.** 2808 cu. in., $6\sqrt[3]{13}$ in. **9.** 1.05. **10.** $35\frac{1}{2}$, 585. **11.** 1 hr. 48 min. **12.** 108. **13.** 3465 cu. ft., $32\frac{2}{5}$ ft. **14.** 35.0. **15.** 2615.2 cu. in. **16.** $10\frac{1}{2}$ in. **17.** $9\frac{1}{3}$ in. **18.** 2.04 mm. **19.** $4511\frac{23}{32}$ approximately. **20.** 60 cu. ft., 71.32 sq. ft. **21.** 569.2 lbs. **22.** 459.6 sq. in. **23.** 1.35 c.c., 371 times. **24.** 3120. **25.** 260.

Exercise IXb. **Page 100.** **1.** 18000. **2.** $754\frac{2}{7}$ sq. ft., $962\frac{1}{2}$ gal. **3.** 6.76 in. **4.** 3.8. **5.** $1244\frac{4}{9}$. **6.** 16 ft. **7.** 7.7 in. **8.** 90 (approximately). **9.** 2611.7 cu. in., 39888. **10.** 2573.3. **11.** 1 : 2 : 3, $(1+\sqrt{2}) : 3 : 4$. **12.** 501.3. **13.** 9.45 in. **14.** 147 : 148. **15.** $808\frac{1}{2}$ cu. in., 360.2 sq. in. **16.** 40 : 36 : 25. **17.** 24000. **18.** $\frac{1}{2}$ litre. **19.** 7 ft. **20.** 2.444 g. **21.** 49.6. **22.** 4589. **23.** 4500 yds. **24.** $23\frac{1}{4}$. **25.** 1 : 2, 1 : 4, 1 : 8. **26.** 256 g.

Exercise Xa. **Page 109.** **1.** 28.1. **2.** 25. **3.** \$17.39. **4.** \$19. **5.** \$34.25. **6.** 4.06 bbls., \$2.31. **7.** \$634.90. **8.** 11, nearly. **9.** \$21 **10.** \$6. **11.** \$24. **12.** \$9.82.

Exercise Xb. **Page 110.** **1.** \$16. **2.** 448. **3.** \$108. **4.** \$1425.60. **5.** 21888 ft. **6.** \$95.80. **7.** \$33.60. **8.** \$332.80. **9.** $1366\frac{2}{3}$ ft. **10.** \$11.82. **11.** 16 ft. 6 in. **12.** (a) \$44; (b) \$221.41; (c) \$129. **13.** \$54.75. **14.** \$208.24. **15.** \$41.34. **16.** 4032. **17.** 480 ac., 1600 rods, 2 miles by roadway, 1.414 miles across. **18.** \$17.28.

Review Exercise I. **Page 114.** **1.** \$25.46. **2.** 60.6. **4.** 7920000. **5.** 174000. **6.** 28.28 in. **7.** 652.2 lbs. **8.** 131 lbs. **9.** 4.42. **10.** 3456. **11.** 6000.6. **12.** 2.47. **13.** (1) \$108.36; (2) \$40; (3) \$224. **14.** 68.9 ft. **15.** (1) $643\frac{1}{2}$; (2) $\frac{5}{176}$. **16.** 77.9 cm. **17.** (1) 7.5 sq. m.; (2) $637\frac{1}{2}$; (3) $637\frac{1}{2}$. **18.** 7.87 sq. in. **20.** $19\frac{1}{5}$. **21.** 98 yds. **22.** $\frac{1}{3}$. **23.** 235+. **24.** 17.65 oz. **25.** 40 : 3. **26.** $12\sqrt[3]{1.273}$ in. **27.** 21.9. **28.** 462 sq. yds. **29.** $\frac{2}{3}$. **30.** .408 in. **31.** 59.7. **32.** $40474\frac{1}{2}$. **33.** 4.43 Km. **34.** 180 g., $66\frac{2}{3}$ c.c. **35.** 7 sq. ft. 140.9 sq. in. **36.** (a) .3125; (b) 1.953; (c) 19.53 lbs. **37.** .5 sq. mm. **38.** 40 in. **39.** 1268.6 gal. **40.** 11 ft. 8.8 in. **41.** $\frac{1}{7}$. **42.** (a) 1 : 11664; (b) 1 : 1259712.

Review Exercise II. **Page 120.** **1.** 43 ft. 1 in. **2.** 9900. **3.** \$177.78. **4.** 12919.21. **5.** 1550. **6.** 1259.7 cu. in. **7.** 1452 sq. in. **8.** 6000, 1201.5 sq. ft. **9.** 80 rds.×48 rds. **10.** \$225. **11.** 7.07 ft. **12.** 102 hrs. 40 min. **13.** 1.458 in. **14.** 113 mm. **15.** 6.364 in. **16.** $\sqrt[3]{88.259}$ in. **17.** 1.76. **18.** 705.6 cu. in. **19.** 26.845 lbs. **20.** 9.6 lbs. **21.** 192 cu. in. **22.** $\sqrt{\pi} : 2$. **23.** 189 sq. in.

Exercise XIa. **Page 131.** **18.** $33\frac{1}{3}$. **19.** 27.3. **20.** $90\frac{10}{11}$. **21.** 25 lbs. **22.** \$360. **23.** $9\frac{3}{4}$; $14\frac{21}{80}$. **24.** (1) $16\frac{2}{3}$; (2) $30\frac{5}{9}$. **25.** 200. **26.** $14\frac{2}{7}$. **27.** 81. **28.** 214184. **29.** 60. **30.** $14\frac{2}{7}$ oz. **31.** 20 tons. **32.** $2\frac{1}{2}$. **33.** 2% decrease. **34.** \$84000. **35.** (a) 57.2 yds.; (b) 7 in. **36.** $9\frac{1}{11}$. **37.** \$5400.

Exercise XIb. **Page 135.** **1.** $33\frac{1}{3}\%$. **2.** 10. **3.** $66\frac{2}{3}\%$. **4.** $33\frac{1}{3}\%$. **5.** \$130. **6.** 12.15% gain. **7.** 32%. **8.** 25%. **9.** 50. **10.** \$65.45. **11.** 35. **12.** $66\frac{2}{3}$. **13.** \$50.40. **14.** $17\frac{3}{11}$. **15.** No loss or gain. **16.** $6\frac{1}{4}\%$ loss. **17.** 26. **18.** 125. **19.** 12.2. **20.** \$800, \$960. **21.** \$61.33. **22.** 25. **23.** \$3. **24.** \$3500. **25.** \$3840. **26.** $13\frac{1}{4}$. **27.** 1500 marks. **28.** $14\frac{2}{7}$.

Exercise XIc. **Page 139.** **1.** 21.8. **2.** Second, \$14.17. **3.** \$503.74. **4.** \$351.50. **5.** 28%. **6.** 27.1%. **7.** \$67.50, 26.3. **8.** \$130. **9.** $33\frac{1}{3}$. **10.** 31.6%. **11.** \$10. **12.** 26.99%. **13.** \$11.52, 38.9. **14.**

\$18.75. **15.** $6\frac{1}{4}$. **16.** 5000 tons. **17.** \$40, \$46, \$41.40. **18.** $16\frac{2}{3}\%$. **19.** \$450. **20.** \$57.96.

Exercise XId. Page 141. **1.** \$560, \$22960. **2.** \$35625. **3.** \$8000. **4.** \$10404. **5.** \$132. **6.** \$2500. **7.** $2\frac{2}{9}\%$. **8.** \$33600. **9.** \$1.16. **10.** 655 lbs. **11.** \$33.90. **12.** $14662\frac{1}{2}$ bush., 490 M. **13.** \$64. **14.** 6000, 70 cts. **15.** 620. **16.** \$3800. **17.** $1\frac{1}{2}\%$. **18.** \$1333.33, \$45. **19.** \$891.25. **20.** \$11107.44.

Exercise XIIa. Page 152. **1.** 20, 2, \$90. **2.** \$143.78. **3.** $16\frac{2}{3}$ mills. **4.** 40 mills. **5.** \$3200. **6.** \$3910. **7.** \$12.70. **8.** \$121.65. **9.** $13\frac{1}{3}$ mills. **10.** 1.52 mills. **11.** 61.3%. **12.** \$6.08. **13.** \$4000. **14.** 18%. **15.** \$4.42. **16.** \$48. **17.** \$3035. **18.** \$68. **19.** \$2.34. **20.** Lowered 10%. **21.** 16 mills. **22.** \$470.90.

Exercise XIIb. Page 156. **1.** \$32. **2.** \$40. **3.** \$535.50. **4.** \$193.20. **5.** \$4000. **6.** \$173.25. **7.** \$55.80. **8.** \$189.

Exercise XIIc. Page 158. **1.** \$19,500. **2.** \$1749.20. **3.** \$230. **4.** \$17020. **5.** \$960.96, \$124.80. **6.** \$441.04. **7.** \$450. **8.** (1) \$306.18, (2) \$1130.63

Exercise XIII. Page 166. **1.** \$4650, \$36. **2.** \$384. **3.** \$110. **4.** \$9456, \$3344. **5.** \$9510.40, \$6329.60. **6.** \$4. **7.** \$32000. **8.** **8.** \$3000, \$2500, \$4500. **9.** \$22.50. **10.** $\frac{37}{40}\%$. **11.** \$26797.50. **12.** \$20000. **13.** \$35.35. **14.** \$2835. **15.** \$1132.80, \$3667.20. **16.** \$61.20. **17.** $1\frac{17}{18}\%$. **18.** 86.8. **19.** Uninsured, \$282. **20.** \$3520. **21.** \$1753.25, \$5450. **22.** $\$2.26\frac{7}{13}$. **23.** $\$10.33\frac{1}{3}$.

Exercise XIVa. Page 177. **1.** (a) \$262.50; (b) \$179.20; (c) \$75; (d) \$1572.90; (e) \$612.50; (f) \$16.18; (g) \$108.57; (h) \$78.43. **2.** (a) \$12.91; (b) \$66.74; (c) \$45.03; (d) \$158.36; (e) \$421.81; (f) \$148.76. **3.** \$44. **4.** \$906.56. **5.** 146 days. **6.** $7\frac{1}{2}\%$. **7.** \$511.94. **8.** \$6272. **9.** 10.02. **10.** \$439000. **11.** \$481.99. **12.** \$1289.12. **13.** \$2000. **14.** \$12000. **15.** $8\frac{1}{3}$. **16.** $12\frac{1}{2}$ yrs. **17.** \$6193.55.

Exercise XIVb. Page 180. **1.** (a) \$4.35, \$535.65; (b) \$11.01, \$738.99; (c) \$17.70, \$1442.30. **2.** (a) 4th Dec. 1921, 109 dys. \$26.88, \$1473.12; (b) 3rd Mar. 1923. 91 dys, \$74.79, \$3925.21; (c) 12th Apl. 1923, 63 dys, \$40.32, \$2879.68; (d) 11th June 1924, 57 dys, \$92.59, \$7319.01; (e) 27th Nov. 1921, 63 dys, \$62.89, \$4491.48. **3.** \$510.52. **4.** \$474.11. **5.** 8.08%. **6.** $7\frac{1}{2}\%$. **7.** 6%. **8.** 90 day. note, \$9.77. **9.** \$32. **10.** \$799.39. **11.** (a) $\frac{189}{18250}$, (b) $\frac{16}{1825}$, (c) $\frac{16}{1809}$. **12.** \$116.98.

Exercise XIVc. Page 183. 1. (a) \$2.10; (b) \$22.43; (c) \$2.39; (d) \$254.19; (e) \$8.43; (f) \$19.88; (g) \$19.40. **2.** (a) \$6.36; (b) \$3.83; (c) \$3.96; (d) \$29.53; (e) \$14.88; (f) \$124.17; (g) \$9.98. **3.** (a) \$501.48; (b) \$1274.88; (c) \$5613.46; (d) \$5615.30; (e) \$5614.38; (f) \$5616.22; (g) \$597.75; (h) \$5951.62. **4.** (a) \$992.99; (b) \$2071.46; (c) \$3352.54; (d) \$6848.41; (e) \$860.69. **5.** (a) \$1494.60; (b) \$2457.53; (c) \$2922.76; (d) \$5271.07. **6.** \$16448.24, \$16620.36.

Exercise XVa. Page 194. 1. \$1928, \$2892, \$4820. **2.** A \$1400, B \$2240, C \$1960. **3.** Jones \$160, Smith \$40. **4.** \$8100. **5.** \$4200, \$6300. **6.** A \$1355.56, B \$2033.33, C \$2711.11. **7.** Brown \$1582, Jones \$791. **8.** \$630, \$2184, \$1386. **9.** A \$1500, B \$1200, C \$560. **10.** \$1000, \$3000. **11.** Wilson \$1260, Stevens \$1296. **12.** A \$3000, B \$3960, C \$6360. **13.** Brown \$2925, Reid \$2437.50, Miller \$877.50. **14.** Johnston \$3186, Wilson \$2544. **15.** A \$22059.72, B \$12990.28. **16.** Brown \$8400, Jones \$7500, Smith \$4500, \$6. **17.** A \$2666.67, B \$3333.33. **18.** Neal \$22800, Colter \$22600, Barton \$17400.

Exercise XVc. Page 199. 1. \$932.63. **2.** \$2282. **3.** 7.46%. **4.** \$5090.62. **5.** \$227.50. **6.** \$3035. **7.** Loan at 6%. **8.** 100. **9.** \$10. **10.** (a) 160; (b) $68\frac{3}{4}$; (c) 75. **11.** Stock \$19.20. **12.** \$27086.25 **13.** $14\frac{2}{7}$% increase. **14.** \$106666.67. **15.** Latter, \$18. **16.** Increase, \$6. **17.** \$5086.25. **18.** $103\frac{1}{12}$. **19.** 32. **20.** \$1259.70. **21.** 2.67%. **22.** \$21840, \$13020. **23.** \$850000. **24.** \$1050. **25.** \$1703.33. **26.** $22\frac{2}{9}$%. **27.** $9\frac{1}{2}$%. **28.** \$5,000,000. **29.** Preferred 10%, Common 12%.

Exercise XVIa. Page 208. 1. \$1135.94. **2.** \$686.45. **3.** \$1448.75 **4.** (a) \$1485.97; (b) \$1247.54; (c) \$727.14. **5.** \$886.73. **6.** \$1626.59. **7.** \$258.44. **8.** \$674.56. **9.** \$367.96. **10.** \$1312.22.

Exercise XVIb. Page 210. 7. $3\frac{7}{15}$ mo. **8.** 5 mo. **9.** $15\frac{9}{11}$ mo. **10.** 29th July. **11.** $12\frac{2}{3}$ mo. **12.** 14th Oct.

Exercise XVIIa. Page 219. 1. \$382.03. **2.** \$1169.86. **3.** \$1689.20. **4.** \$296,195.20. **5.** \$477.54. **6.** Less by \$25.26. **7.** \$3000. **8.** \$1677.50. **9.** $\frac{61}{8000}$. **10.** 7. **11.** 8%, \$1250. **12.** \$137.21. **13.** \$224.06. **14.** 6.09. **15.** 3.92. **16.** \$600.81. **17.** \$994.95. **18.** \$88326.40. **19.** \$1026.77. **20.** \$21600. **21.** \$5609.27

Exercise XVIIb. Page 222. 1. \$2403.85. **2.** (a) \$120; (b) \$839.62; (c) 3050.48; (d) \$2358.28. **3.** \$1901.28. **4.** \$685.71.

5. Cash by \$1.96. **6.** \$2577.10. **7.** 6%. **8.** \$2500. **9.** A's \$39.47 better than B's, \$67.05 better than C's. **10.** \$1057.50. **11.** \$3917.65. **12.** 292. **13.** \$1336.51. **14.** \$1422.68. **15.** 102.72. **16.** \$1519.84, \$1643.85, \$1777.99.

Exercise XVIII. Page 232. **1.** (a) \$1443.60; (b) \$12112.50. **2.** \$2358.10. **3.** £1030 18s $6\frac{3}{4}$d. **4.** \$5858.60. **5.** (a) \$520.83. (b) \$753.75. **6.** $\frac{1}{4}$%. **7.** £619 5s. $3\frac{3}{4}$d. **8.** \$347.49. **9.** \$1544.93. **10.** 4.8181. **11.** Disc., 5% per annum. **12.** \$1281.38. **13.** \$1078.82. **14.** \$1938.25. **15.** \$2608.52. **16.** \$6874.58. **17.** £539. 4s. $4\frac{3}{4}$d. **18.** \$1716.46. **19.** Via London 680.93 f. better. **20.** \$1556.80. **21.** \$5090.91.

Review Exercise III. Page 236. **1.** 60, 40. **2.** 95% (nearly). **3.** \$79.20, 12416 lbs. **4.** \$3130, \$3954, \$7916. **5.** \$1482. **6.** \$280, \$3000, \$5120. **7.** $15\frac{1}{2}$, $7\frac{1}{2}$, 14, 37. **8.** \$36. **9.** \$3.$12\frac{3}{4}$. **10.** \$1458.33, \$2041.67. **11.** \$80. **12.** \$2392.19. **13.** \$303.23. **14.** 33 days, 6th October. **15.** \$771.40. **16.** \$671.06. **17.** \$743.56. **18.** \$353.68. **19.** \$2597.36. **20.** 1.86.

Review Exercise IV. Page 238. **1.** \$22100. **2.** \$624. **3.** 12. **4.** $3\frac{3}{5}$ bush. per acre. **5.** \$3.96. **6.** $28\frac{4}{7}$%. **7.** \$6.76. **8.** \$228. **9.** \$3000. **10.** 20%. **11.** $6\frac{2}{15}$%. **12.** \$6, $66\frac{2}{3}$%. **13.** 4. **14.** \$27866.67. **15.** $63\frac{7}{11}$%. **16.** $12\frac{1}{2}$. **17.** \$1440. **18.** 20000 lbs. **19.** 90, \$38.20. **20.** \$212.95. **21.** 475, \$150. **22.** 406. **23.** Increased $4\frac{4}{9}$%. **24.** 27.1. **25.** 28965. **26.** $\frac{7}{10}$ bbl. **27.** $\frac{1}{7}$ gal. **28.** $2\frac{19}{28}$. **29.** 6.35%. **30.** \$2000. **31.** \$52.25. **32.** \$1.20.

Review Exercise V. Page 243. **1.** \$494.84. **2.** \$730. **3.** 35 cts. **4.** \$3125. **5.** \$350. **6.** \$500. **7.** 2000. **8.** \$65.50 loss. **9.** $63\frac{1}{23}$ **10.** \$7203.78. **11.** \$414.72. **12.** 50.325 mills. **13.** \$1750, \$541.67. **14.** \$900, \$185. **15.** 19625 ft. **16.** 6.97. **17.** £1642. **18.** \$1.80. **19.** 20. **20.** $348\frac{4}{9}$, \$17.42. **21.** 12864 lbs., \$92.33. **22.** \$3.80. **23.** Neither. **24.** \$2,400,000. **25.** \$25000. **26.** \$4654.26. **27.** \$33378.38. **28.** 7. **29.** (a) \$1264.15; (b) 13; (c) \$57 increase. **30.** \$5125. **31.** \$37107.60, \$556.61.

Exercise XXa. Page 280. **1.** (a) .30103, .77815, 1.30103, 1.77815, 2.30103, 2.77815, 3.30103, 3.77815, 5; (b) 3.09132, 2.36135, 1.53643, $\bar{1}$.68196, $\bar{3}$.88241; (c) 5.83158, 1.86293, $\bar{1}$.92671, $\bar{3}$.67786. **2.** (a) 8.996, 3, 179.2, .1295; (b) 10000, 7808, .03977, .1034; (c) 17.926, 399.62, .000050054, 8.9064. **3.** (a) 8.594, 14.06, 5.599;

(b) 274900, 6829, 8.032: (c) 3.307, 28730; (d) .5113, 25.01, 2.911. **4.** (a) 64.883; (b) 2.5266, 230.16, 11.505, .2; (c) 30.418. **5.** (a) 4.0204; (b) .035395; (c) 1.3238.

Exercise XXb. Page 281. **1.** 27.19. **2.** 12284. **3.** 116.35 yds. **4.** (a) 302.4 (cu. in.), 12076 (c.c.), 7270.8 (cu. in.). **5.** 9.11 in. **6.** (a) 1838.7 cu. in.; (b) (1) 6.2 cm., (2) 18.36 in., (3) 1.44 in. **7.** (a) 661.85; (b) 92.11 lbs. **8.** (1) 6.451; (2) 16.39. **9.** 2.205. **10.** 28.89. **11.** 235 sq. cm. **12.** 55.5. **13.** 24855. **14.** 7.44 in. **15.** (1) 694.4; (2) .9143; (3) 1803; (4) .6314; (5) 2.541; (6) 137.16.

Exercise XXc. Page 282. **1.** 5.56 sec. **2.** $1954.70. **3.** 15.59 ac. **4.** $2989. **5.** $644.23. **6.** 10 yrs. **7.** 5 yrs. 9 mo. **8.** $5963, $6710, $14167. **9.** 16.3. **10.** 29687 lbs. **11.** 9.28. **12.** 2.0119 sec. **13.** 7362×10^{16} tons. **14.** $9142.40. **15.** 53. **16.** $2006.90, $4663.20, $5869.90.

Edward Jefferson

910-2nd Ave N.E.

Calgary

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C.H.C.I.

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